Computation by Measurement

*

from quantum optics to foundations

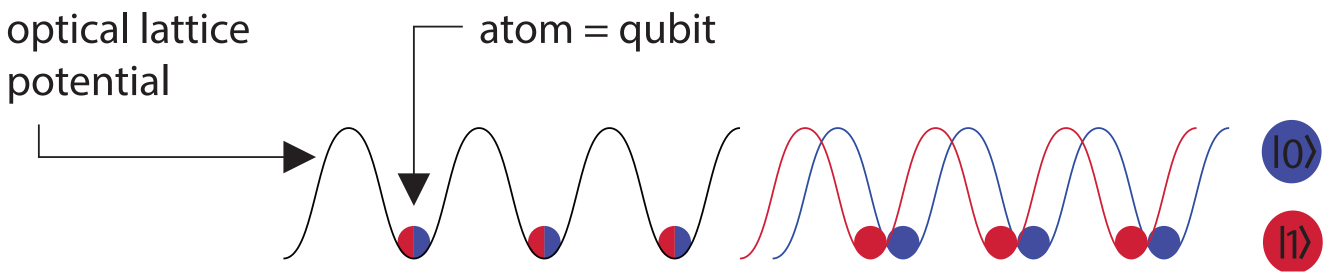
March 15, 2013

Robert Raussendorf, UBC Vancouver

Skizze für die Neujahrskarte 2004.

New-Year's card for 2004

Start: Cold atoms in optical lattices



- Employ state-dependent transport
- Employ state-dependent cold controlled collisions

$\Rightarrow$ translation-invariant Ising interaction among qubits

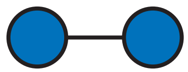
- Knobs to turn: interaction length, duration

  We chose: interaction phase $= \pi$, *nearest-neighbor.*

Q: How can optical lattices be used in quantum information?

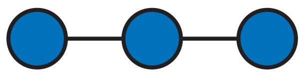
Cluster states

Recall: Ising interaction with interaction phase $= \pi$.



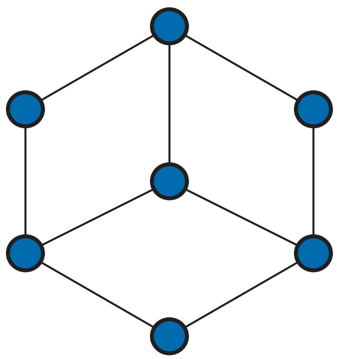
$$|\psi\rangle_2 = |0\rangle_1|+\rangle_2 + |1\rangle_1|-\rangle_2$$

Bell state



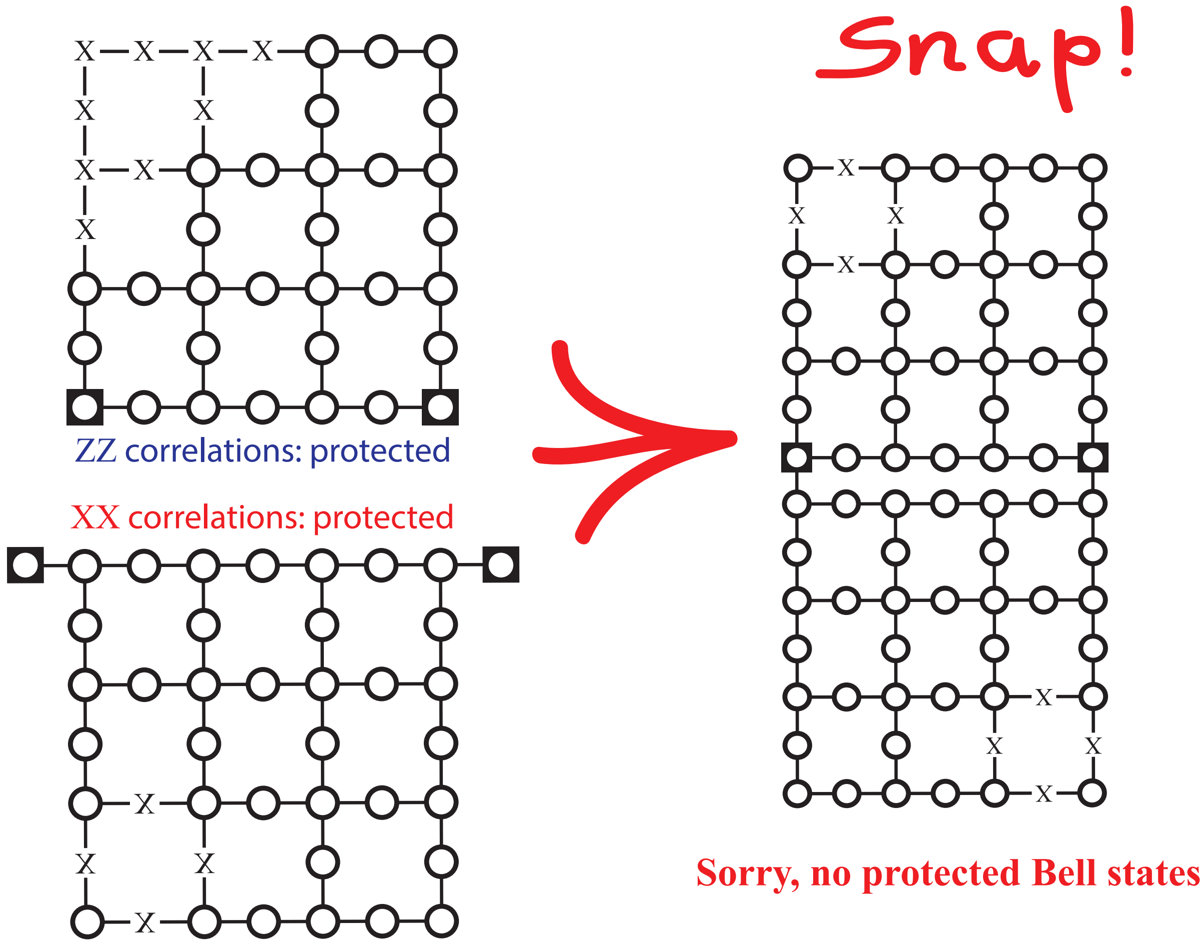
$$|\psi\rangle_3 = |+\rangle_1|0\rangle_2|+\rangle_3 + |-\rangle_1|1\rangle_2|-\rangle_3$$

GHZ-state

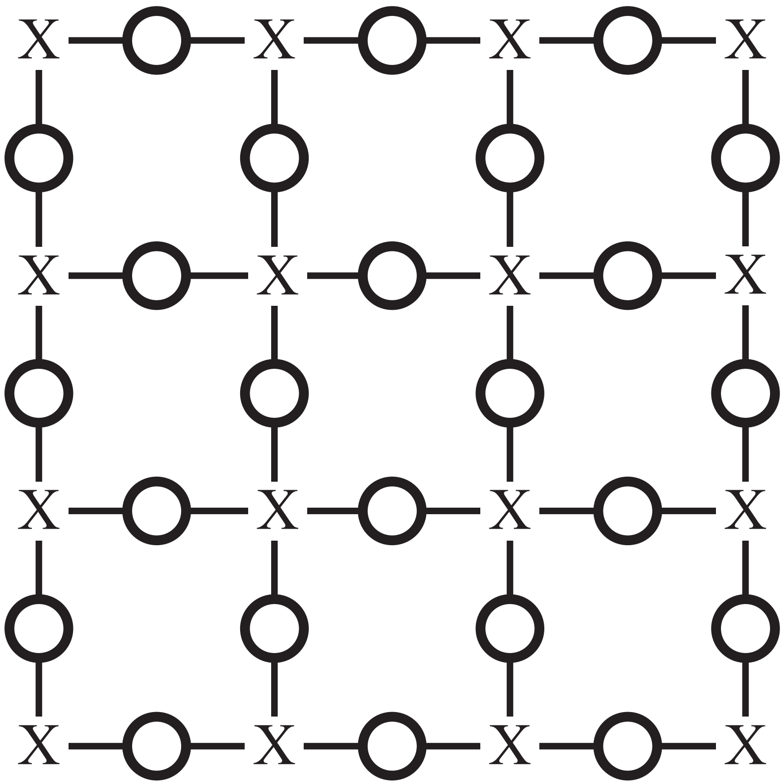


Codeword $\overline{|0\rangle}$ of the Steane quantum code.

Protected transmission line for qubits?



A surface code

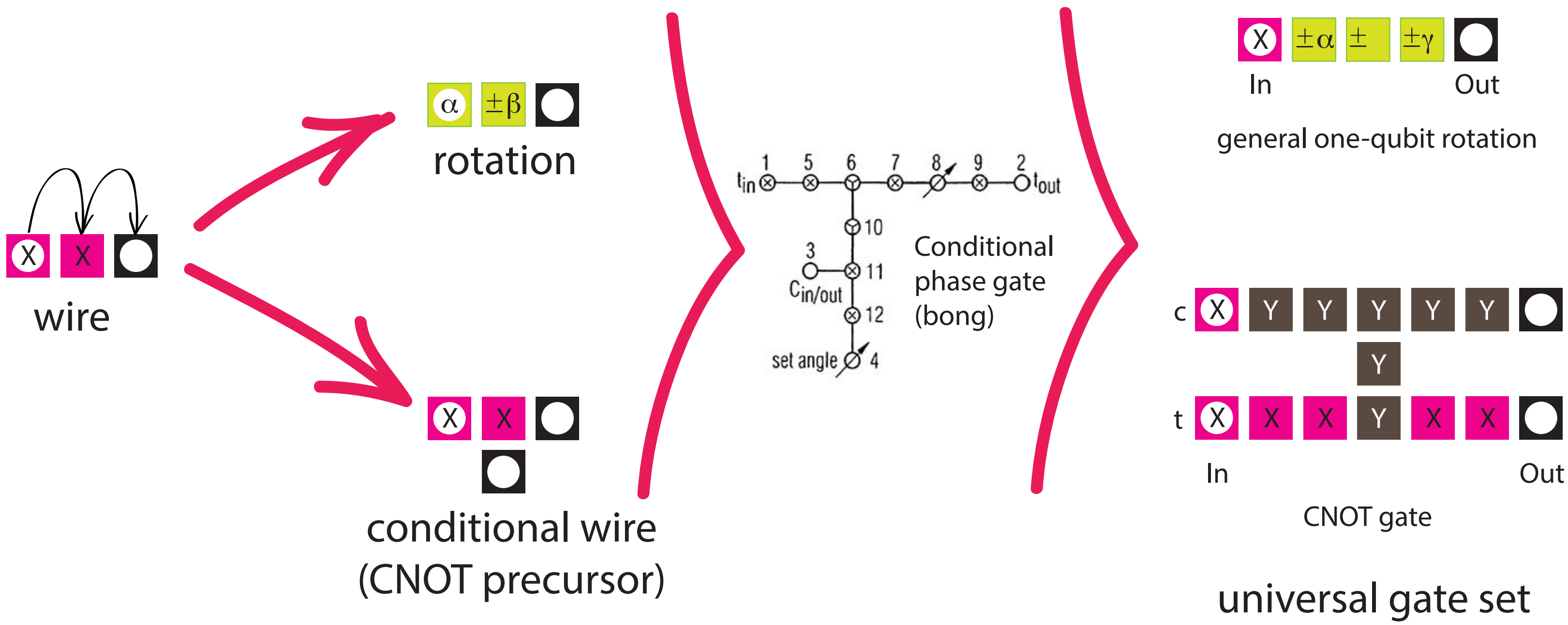


Cluster states
and
surface codes
are
closely related!

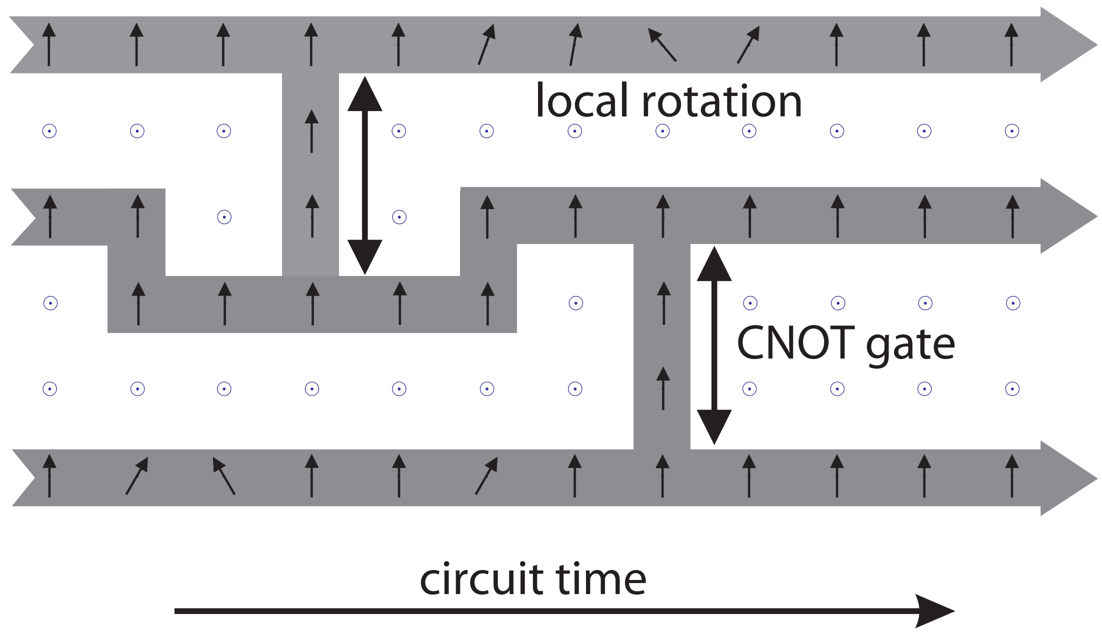
State of unmeasured qubits is a surface code state.

Surface codes: A. Kitaev, Ann. Phys. (N.Y.) 303, 2 (2003); quant-ph/9707021.

Computation by measurement: genealogy of gates



Quantum computation by measurement



- Information written onto a cluster state, processed and read out by one-qubit measurements only.
- The resulting computational scheme is *universal*.

R. Raussendorf and H.-J. Briegel, PRL 86, 5188 (2001).

Computational phases of quantum matter

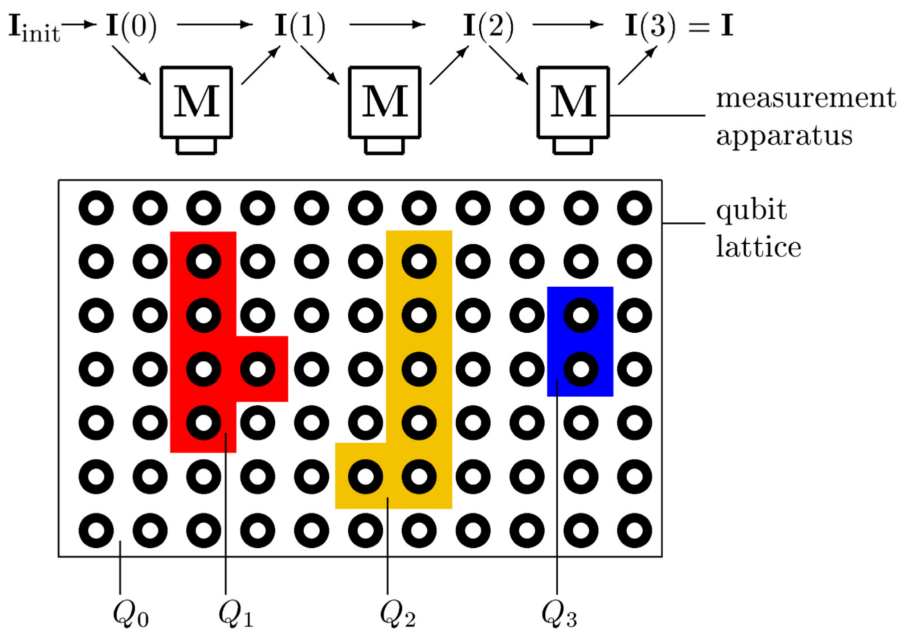
- universal resource states exist
- 2D AKLT states are universal
- surface code states are not universal
- extended universal phase around the AKLT point

  physical and computational phase transition coincide

- universal resource states are rare
- states can be too little entangled to be useful
- states can be too entangled to be useful

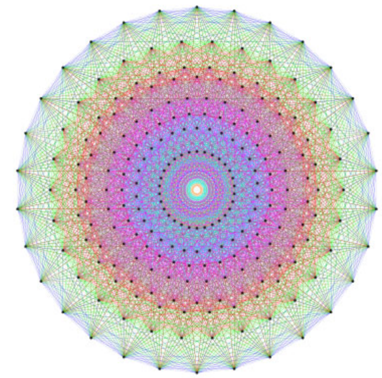
What is the computational power of quantum states?

Computational structures in Hilbert space

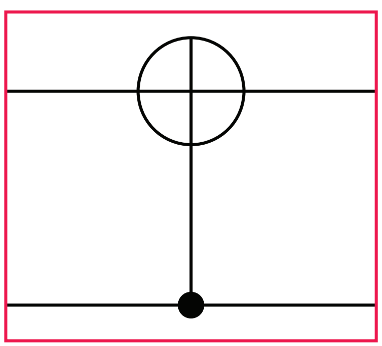


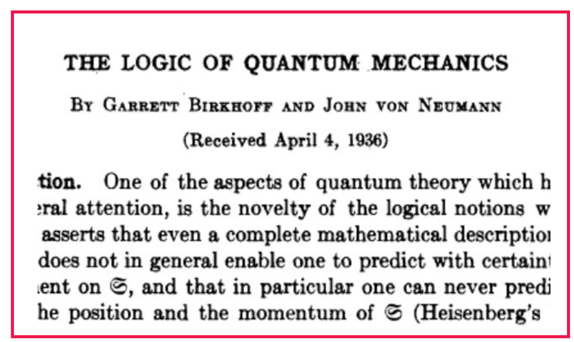
The structure for information processing in MBQC is a quantum-classical hybrid. Its classical part, the information flow vector I

- is initialized with the classical input to the computation,
- ends in the classical output state of the computation,
- in-between governs the adaption of measurement bases.



What computational structures exist in Hilbert space?





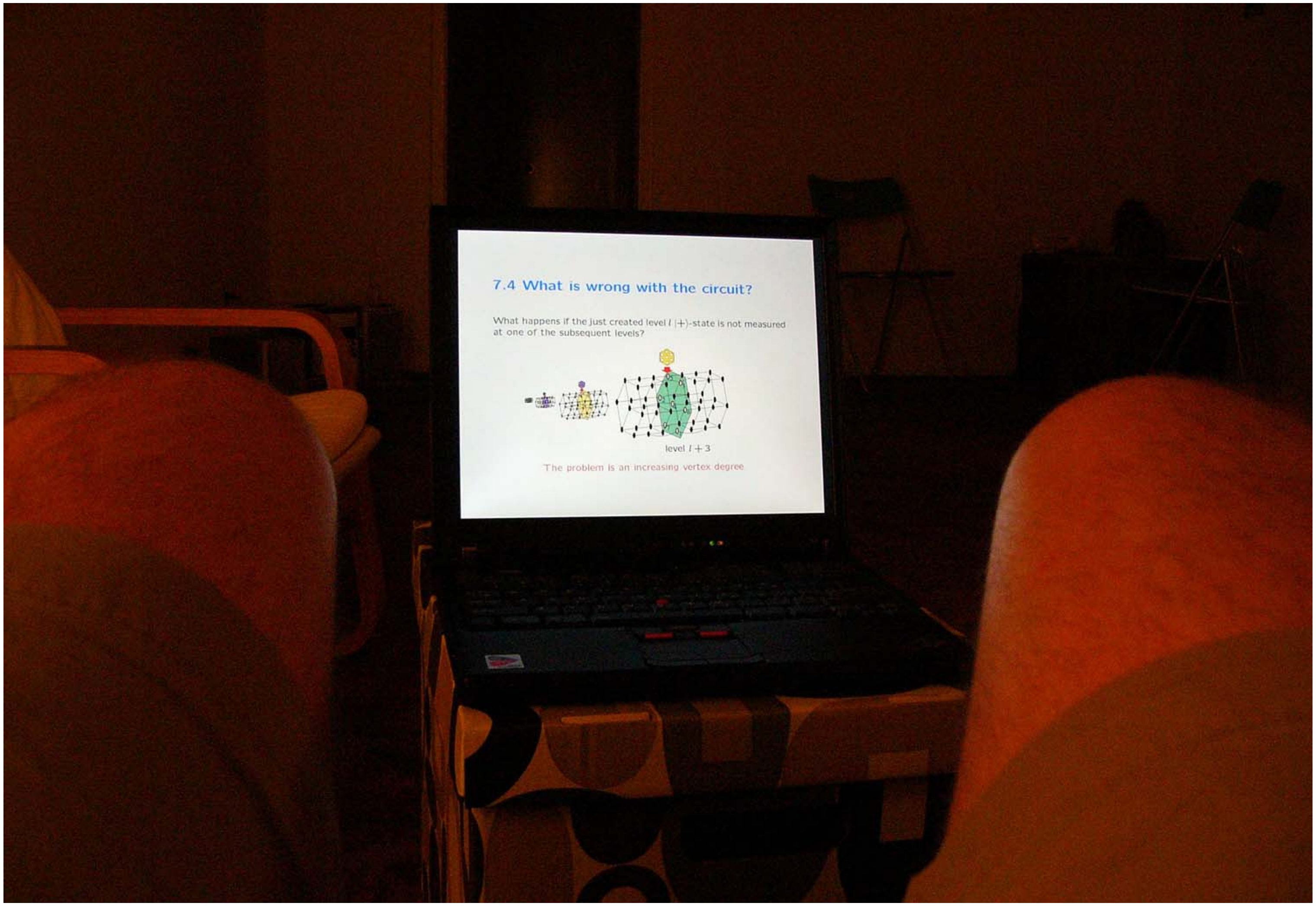
THE LOGIC OF QUANTUM MECHANICS

BY GARRETT BIRKHOFF AND JOHN VON NEUMANN

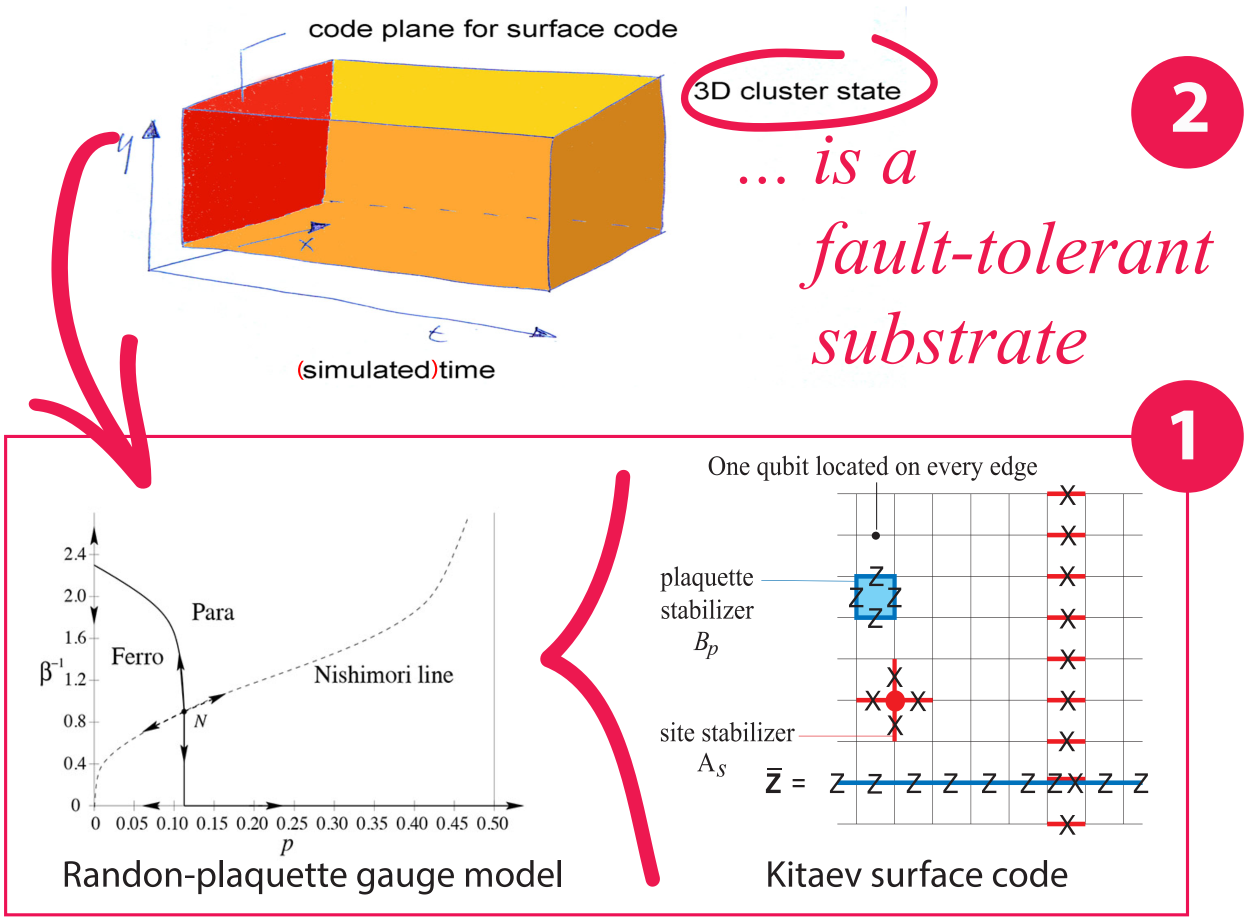
(Received April 4, 1936)

tion. One of the aspects of quantum theory which h
eral attention, is the novelty of the logical notions w
asserts that even a complete mathematical descriptio
does not in general enable one to predict with certain
ent on 𝔖, and that in particular one can never predi
he position and the momentum of 𝔖 (Heisenberg's

The real 2004



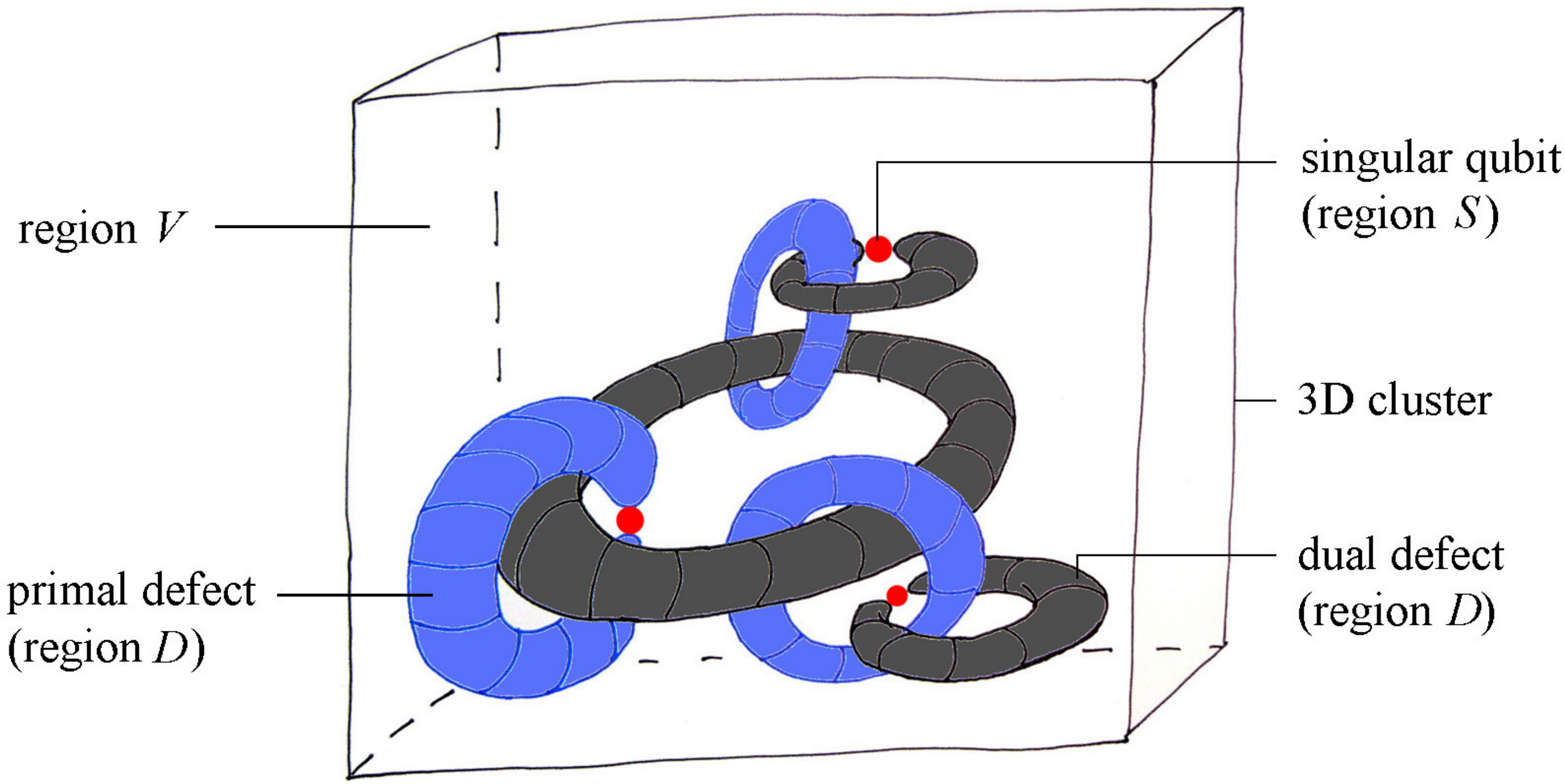
The real 2004



[1] E. Dennis, A. Kitaev, A Landahl, J. Preskill, J. Math. Phys. (N.Y.) **43**, 4452 (2002).

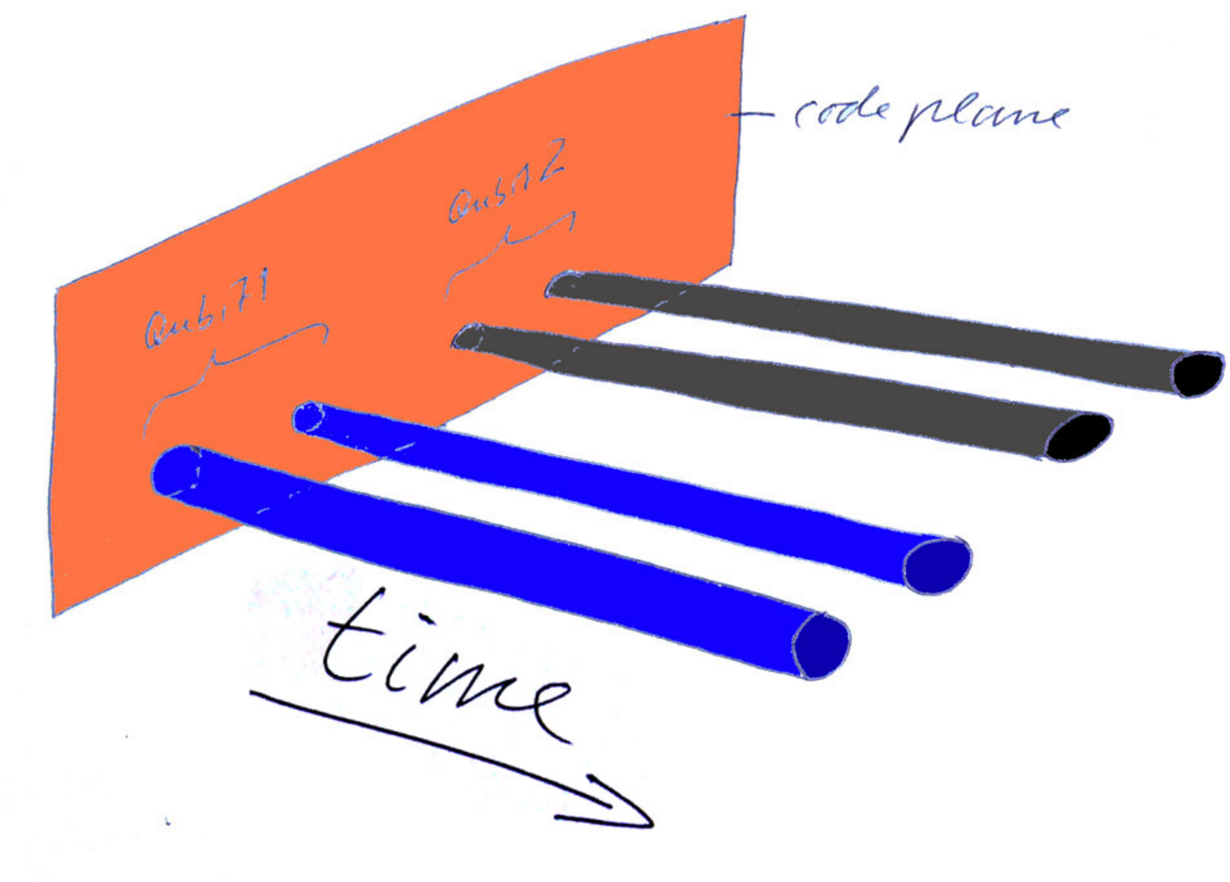
[2] R. Raussendorf, S. Bravyi, J. Harrington, Phys. Rev. A **71**, 062313 (2005).

Fault-tolerant cluster state computation



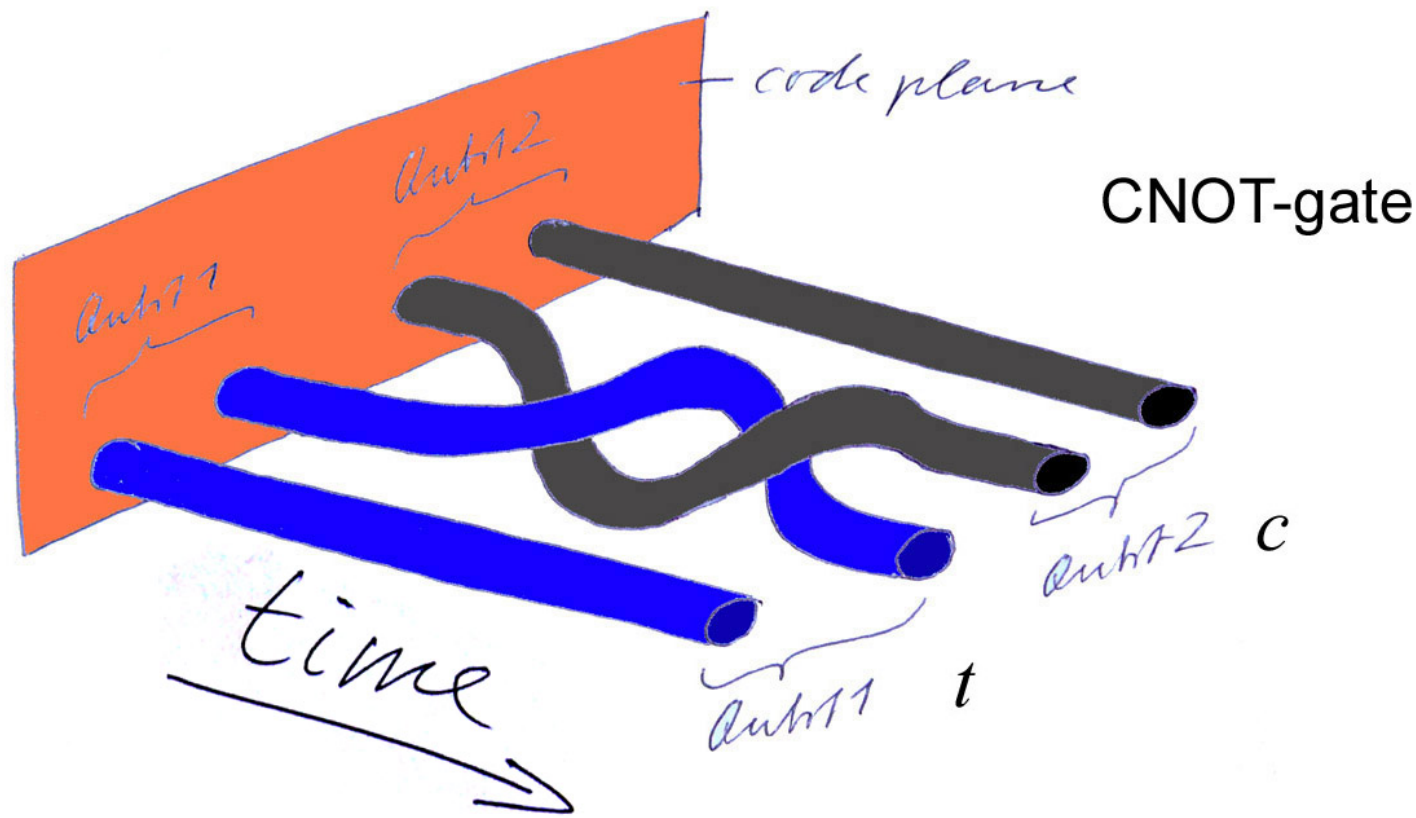
To compute, drill holes into the cluster!

Fault-tolerant cluster state computation



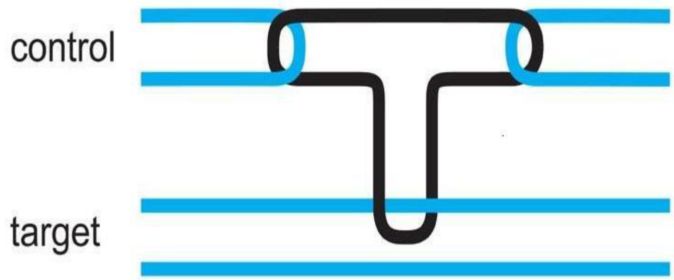
Now consider worldlines of holes.

Fault-tolerant cluster state computation

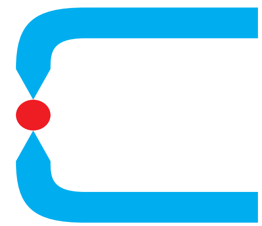


Topological quantum gates are encoded in the way worldlines of primal and dual holes are braided.

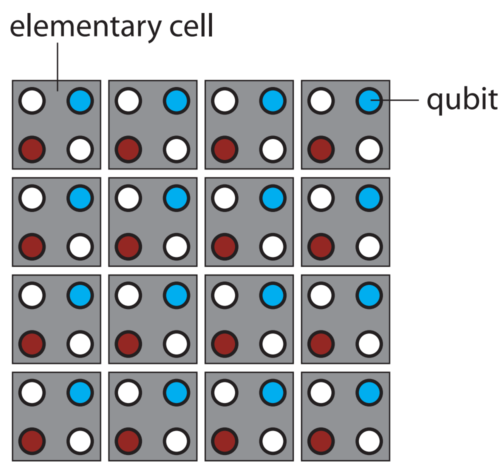
Fault-tolerant cluster state computation



Topological CNOT gate



Magic state distillation

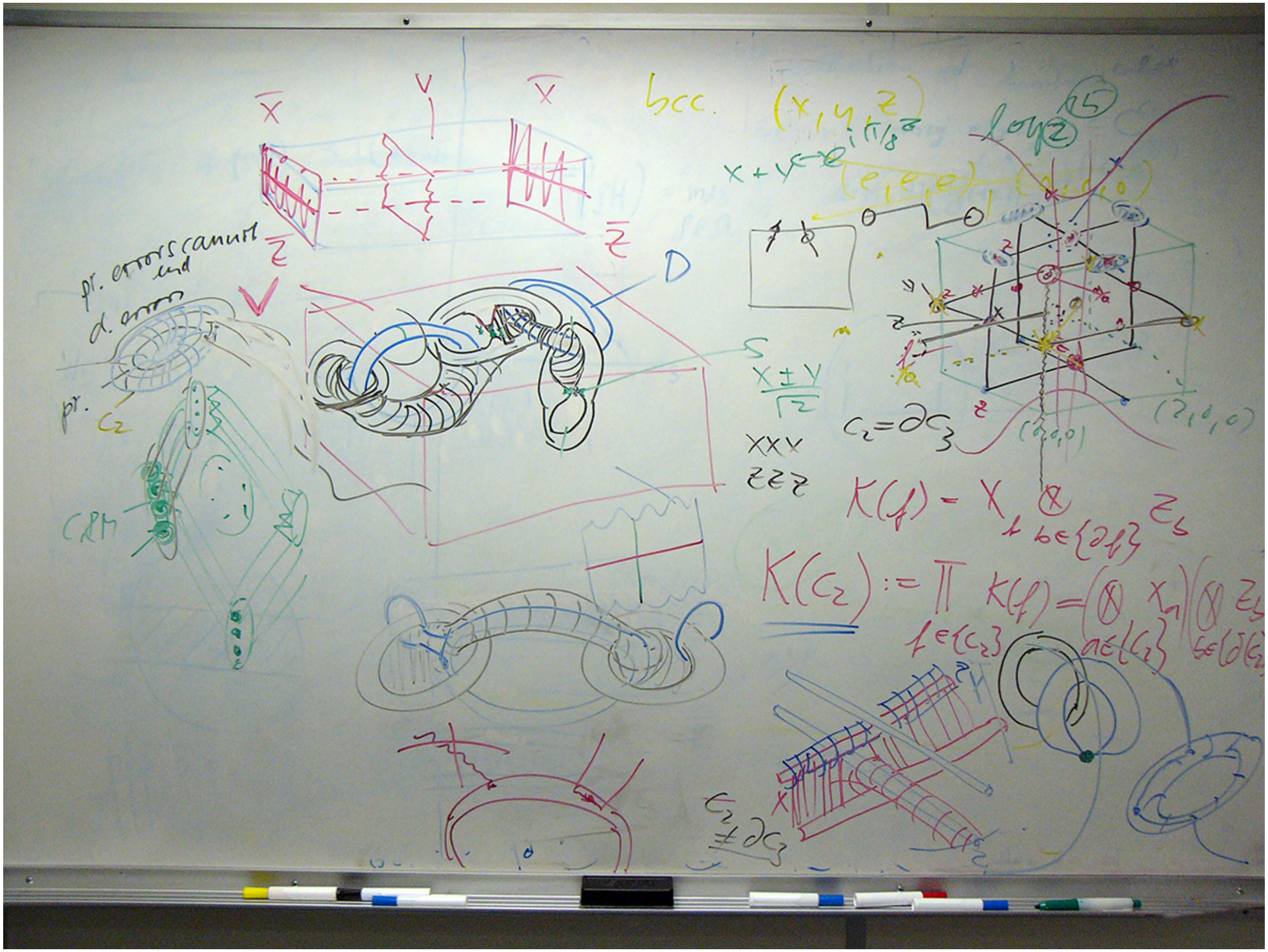


3D cluster => 2D circuit

0.7%

Error threshold

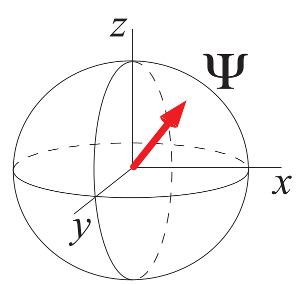
R. Raussendorf and J. Harrington, Phys. Rev. Lett. 98, 190504 (2007).



pr. errors cannot end
d. error
pr.
C2
CRM
bcc.
(x, y, z)
D
S
X ± Y / √2
XXX
ZZZ
c2 = ∂c3
K(f) = X ⊗ Z_b, b ∈ {∂f}
K(c2) := Π K(f), f ∈ {c2}
c2 ≠ ∂c3

What computational structures exist in Hilbert space?

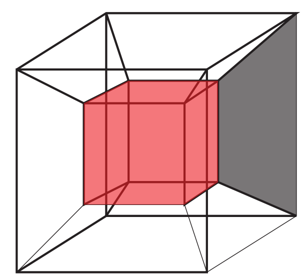
Is quantum computation analog or digital?



$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

... is analog!

(because the set of states is continuous)

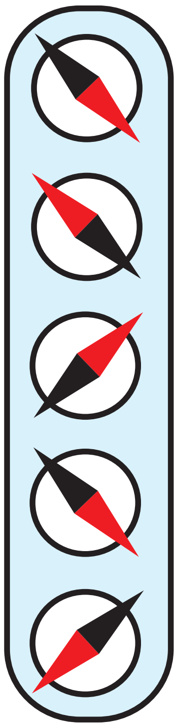


Reed-Muller code

... is digital!

(because the set of transversal encoded gates is discrete)

Quantum codewords compute

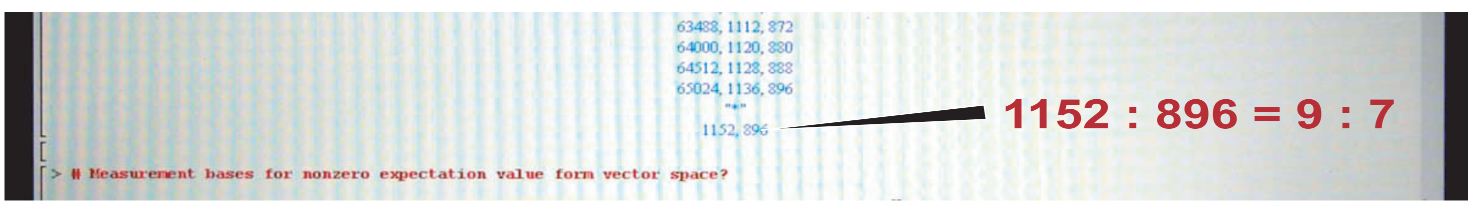


Idea:

Use Reed-Muller code state as computational resource!

* *Classical pre and post-processing is all mod-2-linear (typical)*

* *For 31 qubit code state: mod-2- nonlinear Boolean function deterministically computed*



Contextuality and quantum computation

PRL **102,** 050502 (2009) PHYSICAL REVIEW LETTERS week ending 6 FEBRUARY 2009

Computational Power of Correlations

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(Received 7 May 2008; published 4 February 2009)

We study the intrinsic computational power of correlations exploited in measurement-based quantum computation. By defining a general framework, the meaning of the computational power of correlations is made precise. This leads to a notion of resource states for measurement-based *classical* computation. Surprisingly, the Greenberger-Horne-Zeilinger and Clauser-Horne-Shimony-Holt problems emerge as optimal examples. Our work exposes an intriguing relationship between the violation of local realistic models and the computational power of entangled resource states.

DOI: 10.1103/PhysRevLett.102.050502 PACS numbers: 03.67.Lx, 03.65.Ud, 89.70.Eg

A striking implication of measurement-based quantum computation (MBQC) is that correlations possess intrinsic computational power. MBQC is an approach to computation radically different from conventional circuit models. In a circuit model, information is manipulated by a network of logical gates. In contrast, in the standard model of MBQC (also known as "one-way" quantum computation), information is processed by a sequence of adaptive single-qubit measurements on an entangled multiqubit resource state [1–3]. Impressive characterization of the necessary

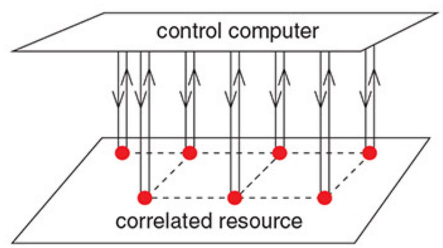


FIG. 1 (color online). The control computer provides one of k choices as the classical input (downward arrows) to each of the

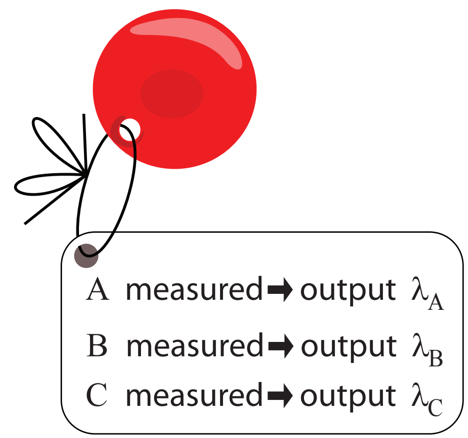
Contextuality of QM

What is a non-contextual hidden-variable model?

quantum mechanics

$$|\Psi\rangle$$

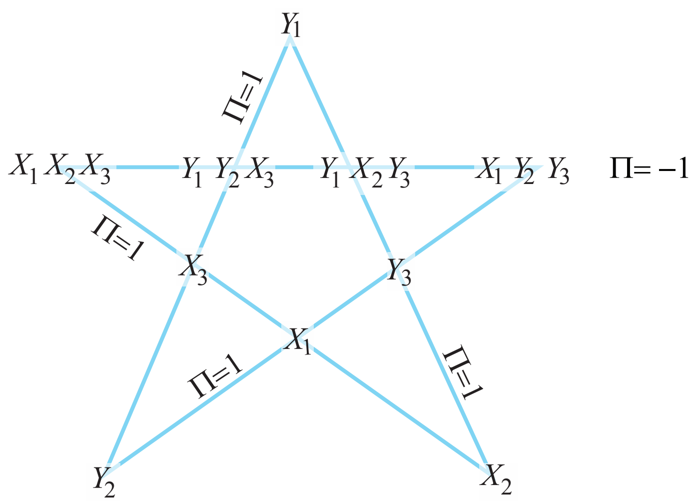
hidden-variable model



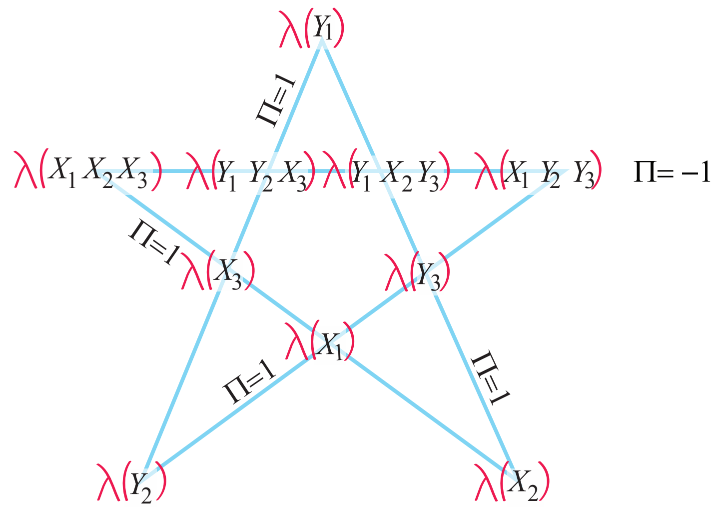
Noncontextuality: Given observables A,B,C: $[A, B] = [A, C] = 0$:
λ_A is *independent* of whether A is measured jointly with B or C.

Theorem [Kochen, Specker]: For dim$(\mathcal{H}) \geq 3$, quantum-mechanics cannont be reproduced by a non-contextual hidden-variable model.

Mermin's proof of the KS theorem in $d = 8$



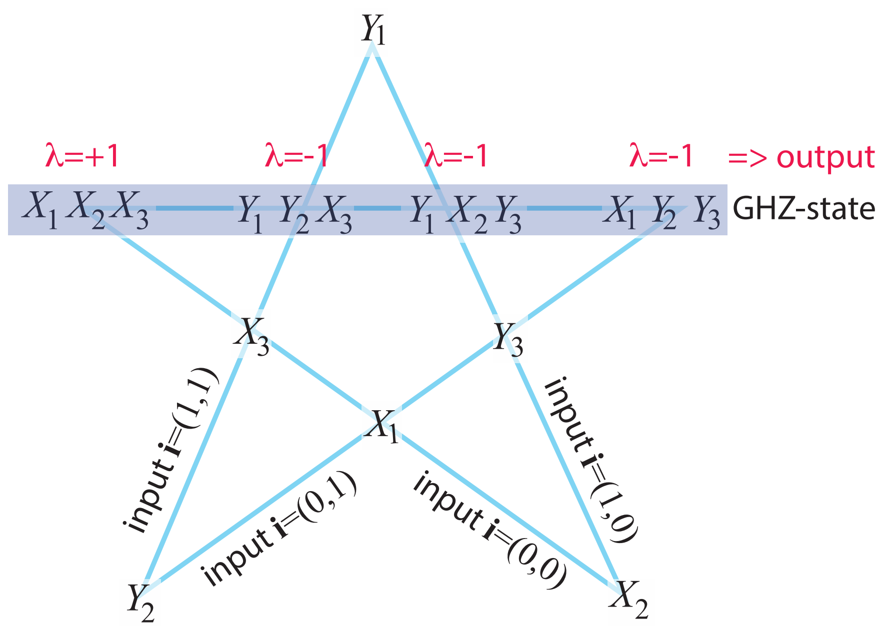
Measurement contexts with Pauli observables on 3 spin 1/2

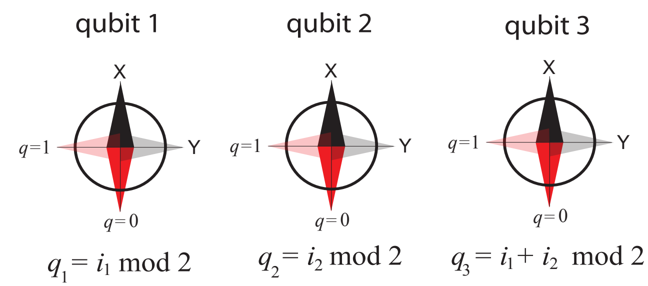


No consistent assignment of values λ(A)=±1 is possible.

- *State-dependent local version:* use GHZ-state $\frac{|000\rangle + |111\rangle}{\sqrt{2}}$. Non-contextuality is founded in locality.

Mermin's KS proof computes!





* Use GHZ state as computational resource

* Compute OR-gate

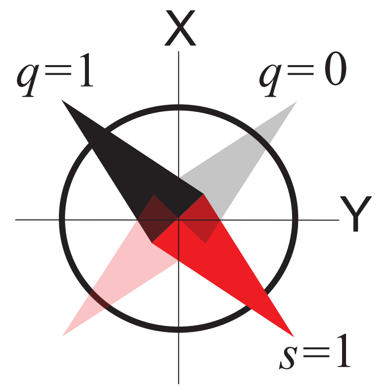
- Classical processing all *linear*, computed OR-gate *non-linear*.

⇒ Classical control computer promoted to classical universality.

J. Anders and D. Browne, PRL 102, 050502 (2009).

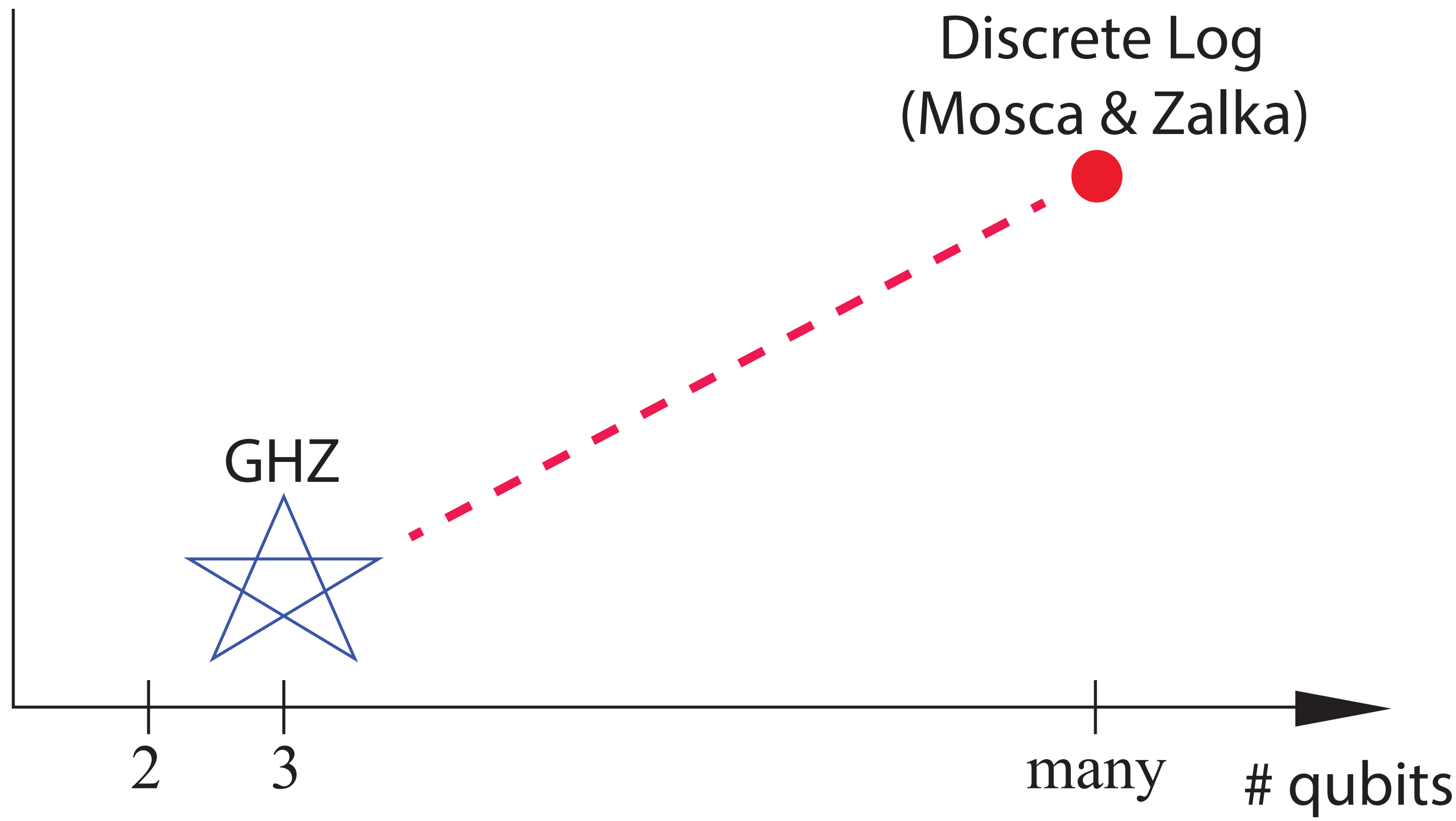
Contextuality vs. non-linearity

Is the link between contextuality and non-linearity general?



Theorem 1: Consider an MBQC with 2 measurement bases and outcomes per local system, in which the classical pre-and post processing is linear. If this MBQC deterministically evaluates a non-linear Boolean function then it is (strongly) contextual.

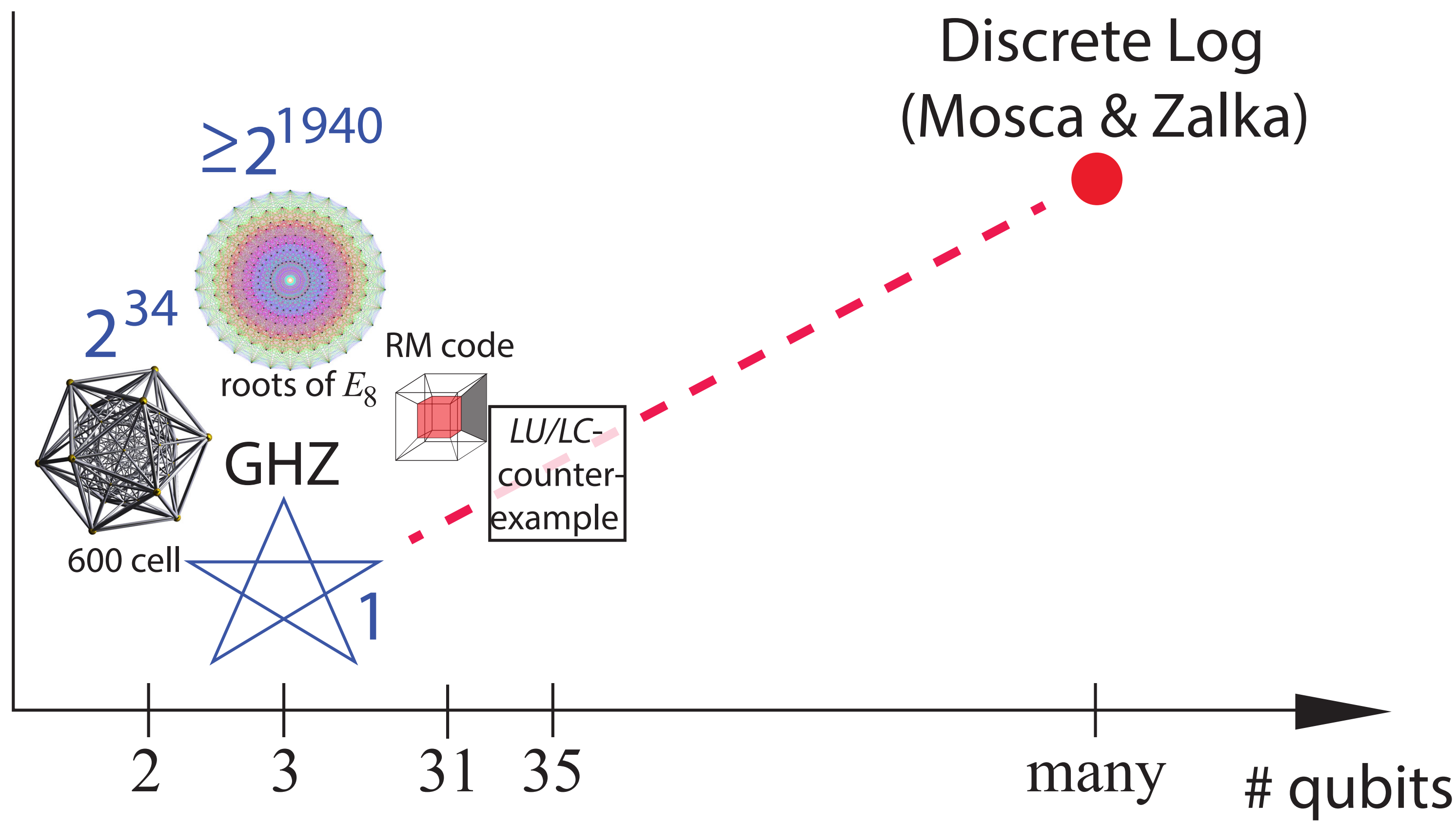
Contextual MBQCs: phenomenology



- M&Z (2003): Discrete Log can be made deterministic.

$\Rightarrow$ By theorem 1, the corresponding MBQC is contextual.

Contextual MBQCs: phenomenology

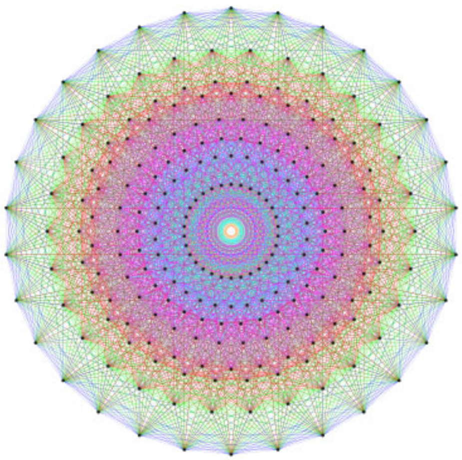


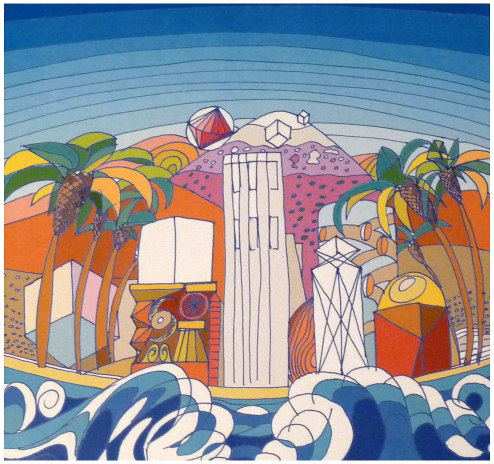
- **Figures in blue:** # of Mermin-type KS proofs

$\Rightarrow$ *Potentially large number of computing structures in HS.*

Summary

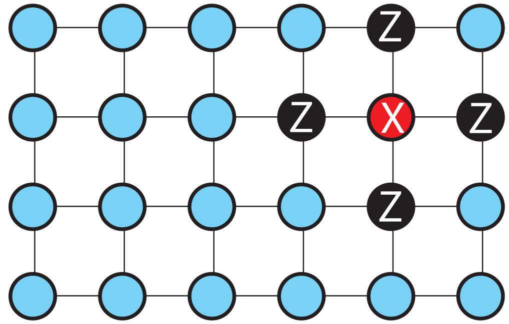
- *Which computational structures exist in Hilbert space?*





No answer yet, but new phenomenology

Cluster states



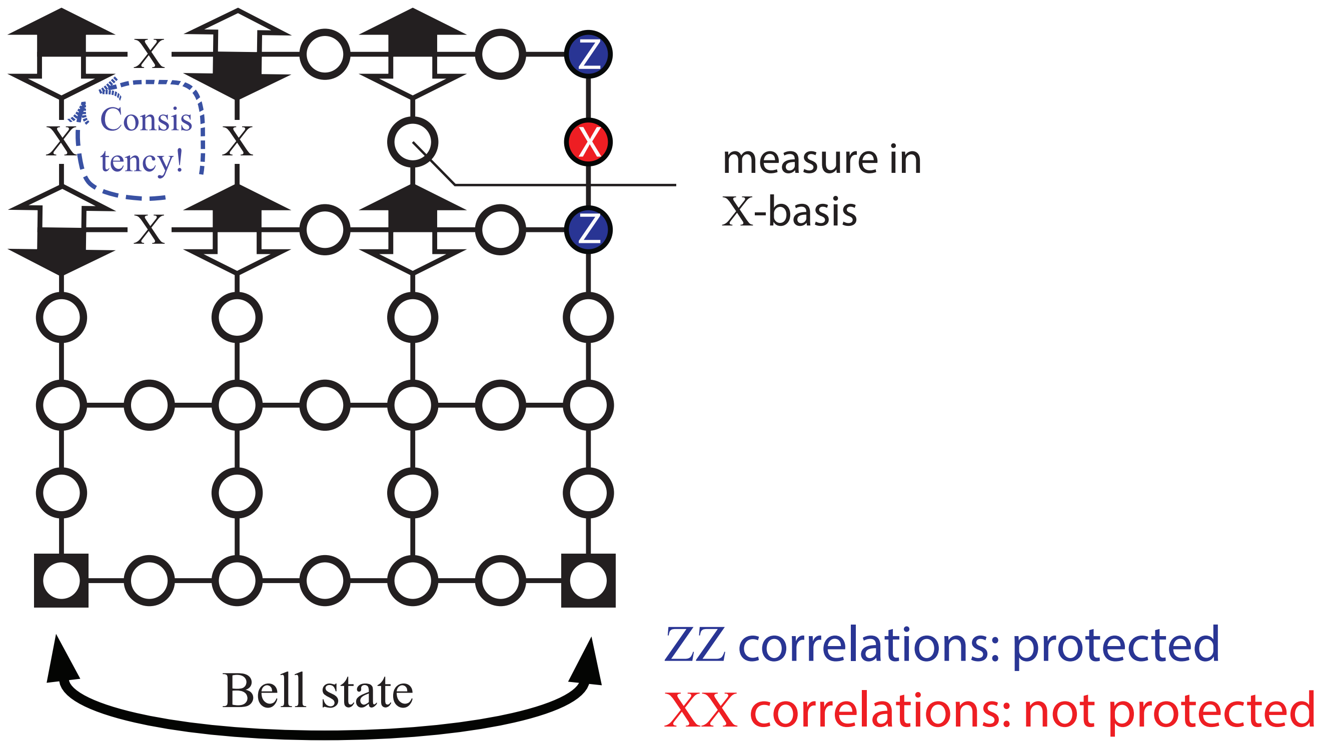
Definition: A cluster state $|\phi\rangle$ associated with a graph G is the single common eigenstate of the stabilizer operators $\{K_a\}$,

$$K_a|\phi\rangle = |\phi\rangle, \ \forall a \in V(G),$$

with

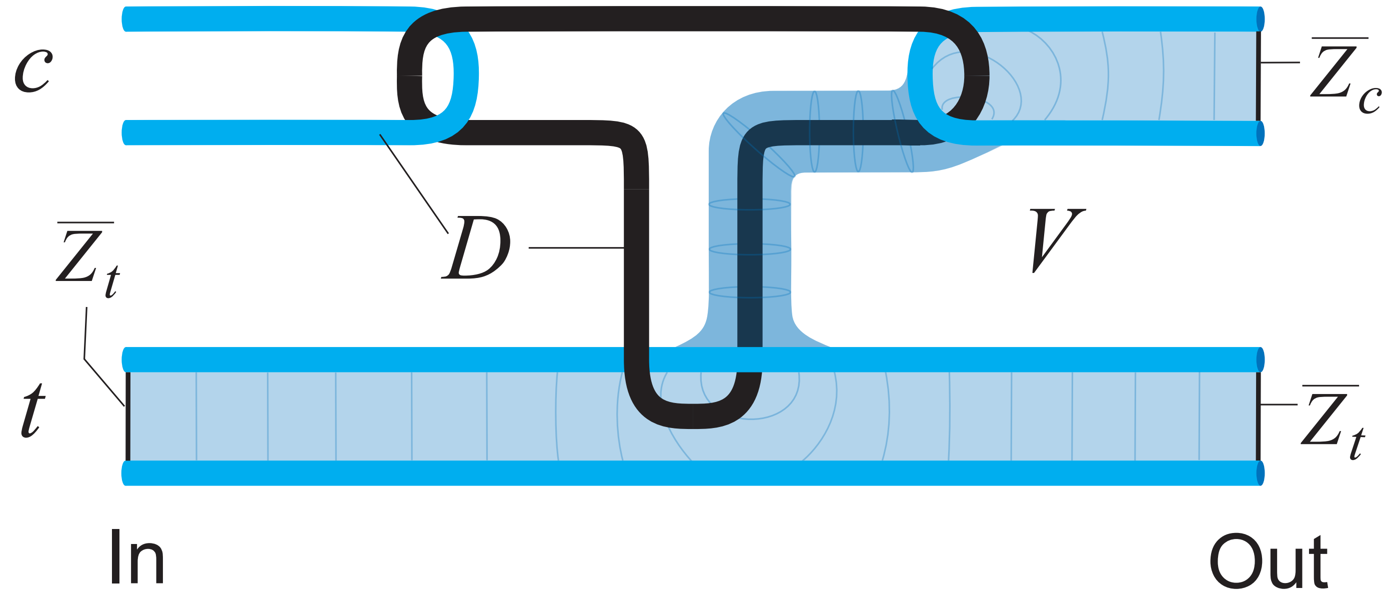
$$K_a = X_a \bigotimes_{b|(a,b)\in E(G)} Z_b, \quad \forall a \in V(G). \tag{1}$$

Protected transmission line for qubits?



Measure all other qubits in the X-basis

Correlations and homology



- *Functioning of gates described by correlation surfaces*
- *Mathematical foundation: relative homology*