Computation by Measurement

* from quantum optics to foundations

March 15, 2013

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Skizze für die Neujahrskarte 2004.
New-Year’s card for 2004
Start: Cold atoms in optical lattices

• Employ state-dependent transport
• Employ state-dependent cold controlled collisions
⇒ translation-invariant Ising interaction among qubits
• Knobs to turn: interaction length, duration

\[ \text{We chose: interaction phase } = \pi, \text{ nearest-neighbor.} \]

Q: How can optical lattices be used in quantum information?
Cluster states

Recall: Ising interaction with interaction phase $= \pi$.

$$|\psi\rangle_2 = |0\rangle_1|+\rangle_2 + |1\rangle_1|-\rangle_2$$

Bell state

$$|\psi\rangle_3 = |+\rangle_1|0\rangle_2|+\rangle_3 + |-\rangle_1|1\rangle_2|-\rangle_3$$

GHZ-state

Codeword $|0\rangle$ of the Steane quantum code.
Protected transmission line for qubits?

Sorry, no protected Bell states!
A surface code

State of unmeasured qubits is a surface code state.

Cluster states and surface codes are closely related!

Computation by measurement: genealogy of gates

- **CNOT gate**
- **General one-qubit rotation**
- **Conditional phase gate**
- **Conditional wire (CNOT precursor)**
- **Universal gate set**
Quantum computation by measurement

- Information written onto a cluster state, processed and read out by one-qubit measurements only.
- The resulting computational scheme is universal.

Computational phases of quantum matter

Universal resource states exist

2D AKLT states are universal

Extended universal phase around the AKLT point

physical and computational phase transition coincide

Surface code states are not universal

States can be too little entangled to be useful

States are rare

States can be too entangled to be useful

What is the computational power of quantum states?
The structure for information processing in MBQC is a quantum-classical hybrid. Its classical part, the information flow vector $I$

- is initialized with the classical input to the computation,
- ends in the classical output state of the computation,
- in-between governs the adaption of measurement bases.
What computational structures exist in Hilbert space?
7.4 What is wrong with the circuit?

What happens if the just created level 1-1-1-1-1-1 is not measured at one of the subsequent levels?

The problem is an increasing vertex degree.
The real 2004

One qubit located on every edge

\[ \bar{Z} = \begin{array} \text{Z} \text{Z} \text{Z} \text{Z} \text{Z} \text{Z} \text{Z} \end{array} \]

is a fault-tolerant substrate


Fault-tolerant cluster state computation

To compute, drill holes into the cluster!
Now consider worldlines of holes.
Fault-tolerant cluster state computation

Topological quantum gates are encoded in the way worldlines of primal and dual holes are braided.
Fault-tolerant cluster state computation

Topological CNOT gate

Magic state distillation

3D cluster => 2D circuit

0.7%
Error threshold

What computational structures exist in Hilbert space?

Is quantum computation analog or digital?

\[ |\Psi\rangle = \alpha |0\rangle + \beta |1\rangle \]

... is analog!
(because the set of states is continuous)

Reed-Muller code

... is digital!
(because the set of transversal encoded gates is discrete)
Idea:

Use Reed-Muller code state as computational resource!

* Classical pre and post-processing is all mod-2-linear (typical)

* For 31 qubit code state: mod-2- nonlinear Boolean function deterministically computed

\[ 1152 : 896 = 9 : 7 \]
Computational Power of Correlations

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(Received 7 May 2008; published 4 February 2009)

We study the intrinsic computational power of correlations exploited in measurement-based quantum computation. By defining a general framework, the meaning of the computational power of correlations is made precise. This leads to a notion of resource states for measurement-based classical computation. Surprisingly, the Greenberger-Horne-Zeilinger and Clauser-Horne-Shimony-Holt problems emerge as optimal examples. Our work exposes an intriguing relationship between the violation of local realistic models and the computational power of entangled resource states.

DOI: 10.1103/PhysRevLett.102.050502

PACS numbers: 03.67.Lx, 03.65.Ud, 89.70.Eg

A striking implication of measurement-based quantum computation (MBQC) is that correlations possess intrinsic computational power. MBQC is an approach to computation radically different from conventional circuit models. In a circuit model, information is manipulated by a network of logical gates. In contrast, in the standard model of MBQC (also known as “one-way” quantum computation), information is processed by a sequence of adaptive single-qubit measurements on an entangled multiqubit resource state [1–3]. Impressive characterization of the necessary...
Contextuality of QM

What is a non-contextual hidden-variable model?

Quantum mechanics    Hidden-variable model

$\Psi$

Noncontextuality: Given observables $A,B,C$: $[A, B] = [A, C] = 0$: $\lambda_A$ is independent of whether $A$ is measured jointly with $B$ or $C$.

**Theorem** [Kochen, Specker]: For $\dim(\mathcal{H}) \geq 3$, quantum-mechanics cannot be reproduced by a non-contextual hidden-variable model.
Mermin’s proof of the KS theorem in $d = 8$

Measurement contexts with Pauli observables on 3 spin 1/2

No consistent assignment of values $\lambda(A) = \pm 1$ is possible.

- **State-dependent local version**: use GHZ-state $\frac{|000\rangle + |111\rangle}{\sqrt{2}}$. Non-contextuality is founded in locality.
Mermin’s KS proof computes!

*$\text{Use GHZ state as computational resource}$

*$\text{Compute OR-gate}$

• Classical processing all $\textit{linear}$, computed OR-gate $\textit{non-linear}$.

$\Rightarrow$ Classical control computer promoted to classical universality.

Contextuality vs. non-linearity

Is the link between contextuality and non-linearity general?

Theorem 1: Consider an MBQC with 2 measurement bases and outcomes per local system, in which the classical pre-and post processing is linear. If this MBQC deterministically evaluates a non-linear Boolean function then it is (strongly) contextual.
  ⇒ By theorem 1, the corresponding MBQC is contextual.
Contextual MBQCs: phenomenology

- Figures in blue: $\#$ of Mermin-type KS proofs

$\Rightarrow$ Potentially large number of computing structures in HS.
Summary

Which computational structures exist in Hilbert space?

No answer yet, but new phenomenology
**Definition:** A cluster state $|\phi\rangle$ associated with a graph $G$ is the single common eigenstate of the stabilizer operators $\{K_a\}$,

$$K_a |\phi\rangle = |\phi\rangle, \quad \forall a \in V(G),$$

with

$$K_a = X_a \bigotimes_{b|(a,b) \in E(G)} Z_b, \quad \forall a \in V(G). \quad (1)$$
Protected transmission line for qubits?

ZZ correlations: protected
XX correlations: not protected

Consistency!

Measure all other qubits in the X-basis

Bell state

ZZ correlations: protected
XX correlations: not protected
Correlations and homology

- Functioning of gates described by correlation surfaces
- Mathematical foundation: relative homology