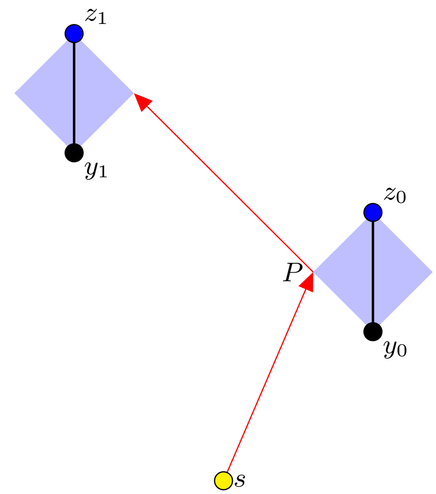
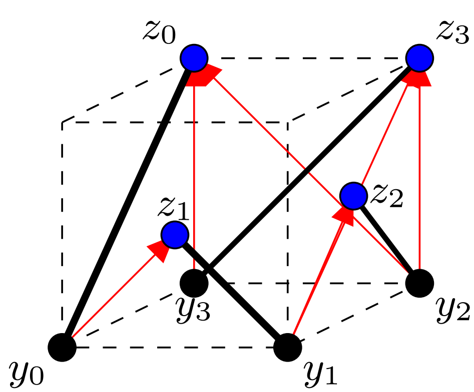
Where and when can a qubit be?





Patrick Hayden
McGill University

Based on arXiv:1210.0913 with Alex May

We’re delighted to celebrate our friend most illustrious,
A scientist brilliant and uncommon industrious.
From quarks to the cosmos, he doesn't dissemble,
He's a storehouse of knowledge, with effort assembled.

He pointed out early the need to be leery
Of the prevailing cosmological theory.
Our universe should have been full to affliction
With magnets diverging against Gauss's prescription.

An expert on particles, fields and forces,
He juggled axions, symmetries, masses (and courses).
But somewhere along the proverbial way,
Quantum computers became his dossier.

For corruption from bit flips all the way to bosonic,
He's invented new gadgets that are just the right tonic.

For Alice and Bob filtering long-estranged Eve,
He proved that entanglement provided the sieve.

And in systems exotic confined to the plane,
He found qubits tangled in quasiparticle skeins.

How fortunate we are this spacetime to share,
And to all wish John Preskill *bon anniversaire*.

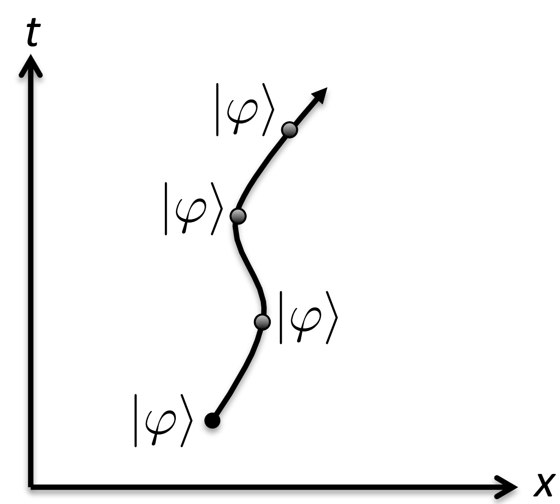
Quantum information bedrock

Quantum information cannot be cloned.

$|\varphi\rangle \not\to |\varphi\rangle|\varphi\rangle$

Quantum information cannot be replicated in space.

And yet…



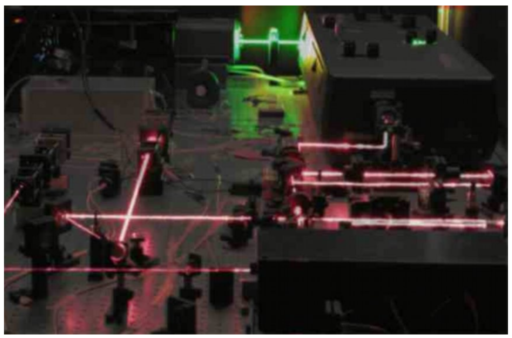
Quantum information **must** be widely replicated in space*time*.

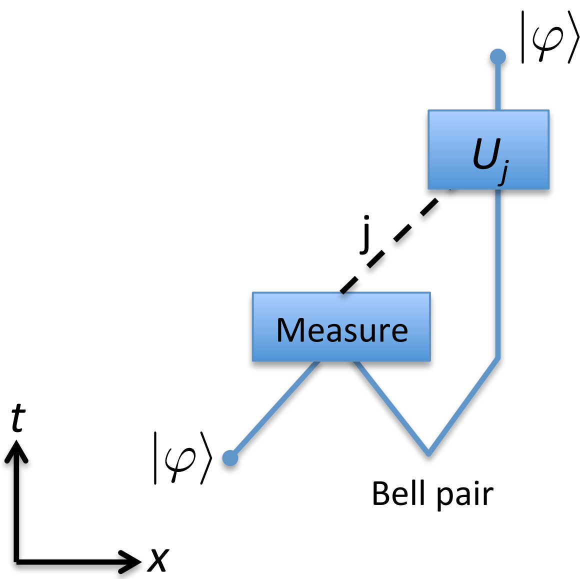
This talk will precisely characterize which forms of replication are possible.

Goal: understand how quantum information can be distributed in space and time

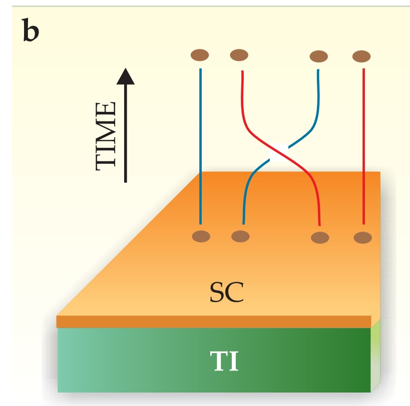
The inadequacy of trajectories

Teleportation





Topological order



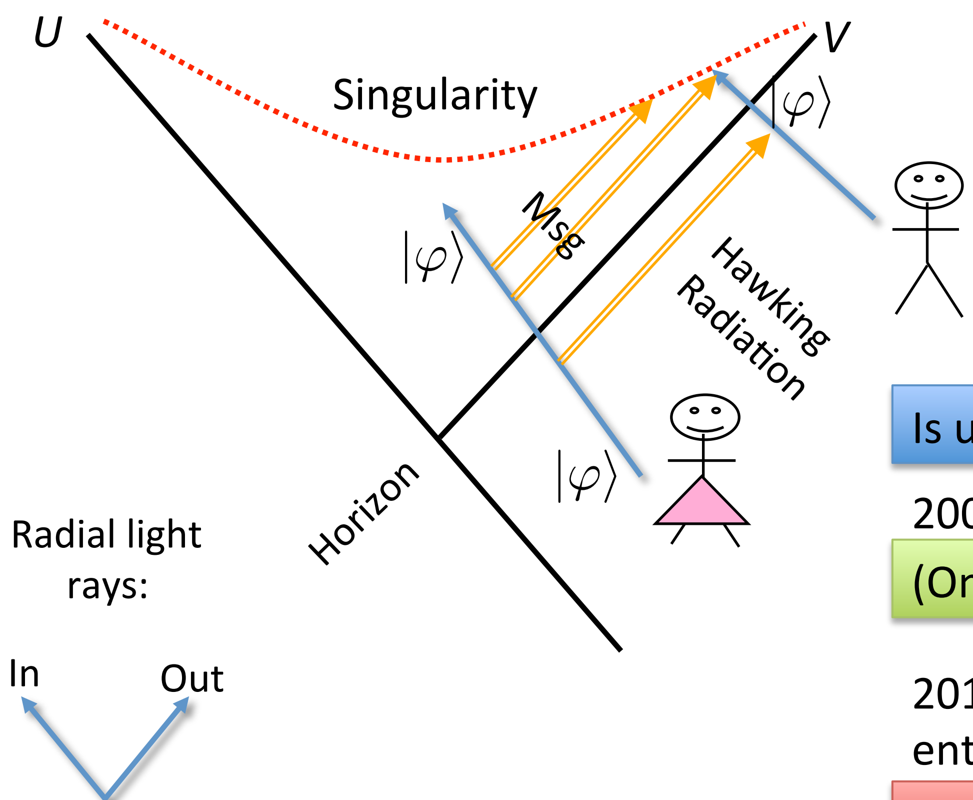
Topologically encoded quantum information is delocalized

In principle robust to local noise

Information replicated in multiple overlapping spatial regions

Figure credits: Gisin group (Geneva) and Qi-Zhang (Stanford)

Cloning, Black Holes and Firewalls



Quantum information *appears* to be cloned

Spacetime structure prevents comparison of the clones (?)

Is unitarity safe?

2007: H & Preskill study *old* black holes.

(Only just) safe

2012: Almheiri et al. consider φ to be entanglement with late time Hawking photon

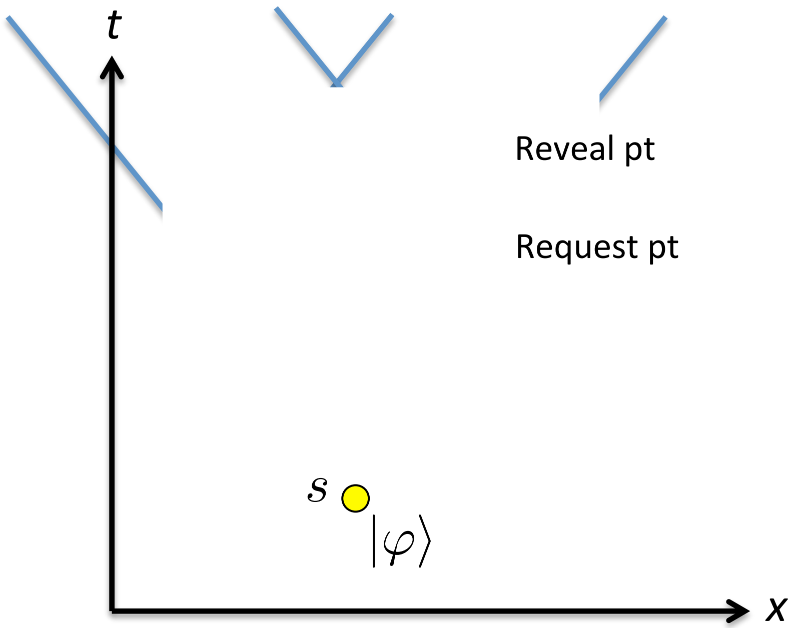
Firewalls!

[Page, Preskill, Susskind 93][Susskind, Thorlacius, Uglum 93]

Outline

- Summoning information in spacetime
  - Where and when is my quantum information?
- Simple examples
- The general case
  - Complete characterization of which spacetime regions can contain the same quantum information
- Conclusions
  - Application to cryptography

Summoning



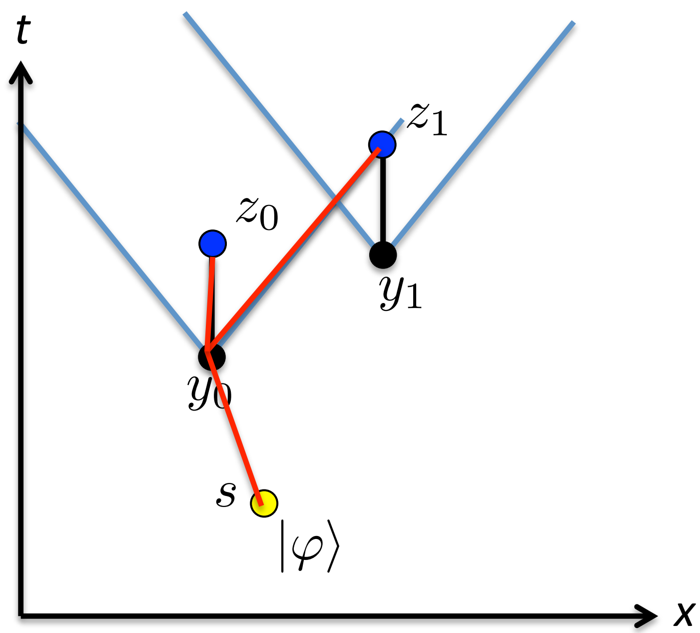
Unknown quantum information is originally localized at s.

A request for the info will be made at either y_0 or y_1, at which point the state must be exhibited at z_0 or z_1, resp.

This is **prohibited** by the combination of no-cloning and relativistic causality if the line segments y_0z_0 and y_1z_1 are outside each others' lightcones.

Kent : arXiv:1101.4612

Summoning



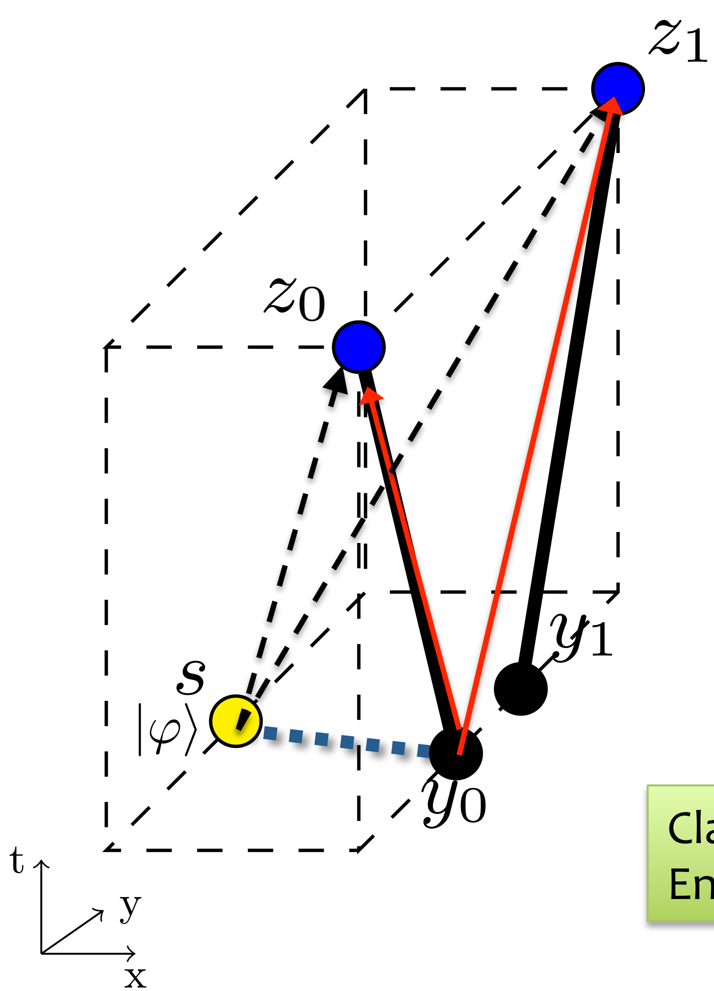
Unknown quantum information is originally localized at s.

A request for the info will be made at either y_0 or y_1, at which point the state must be exhibited at z_0 or z_1, resp.

This is possible if… ?

Summoning is possible iff z_1 is in the future of y_0 or z_0 is in the future of y_1.

Teleportation: A nontrivial example



Request arrives at y_0 or y_1

Direct transmission of φ from s to the correct z_0 or z_1 is impossible

Instead, a Bell pair is shared between s and y_0

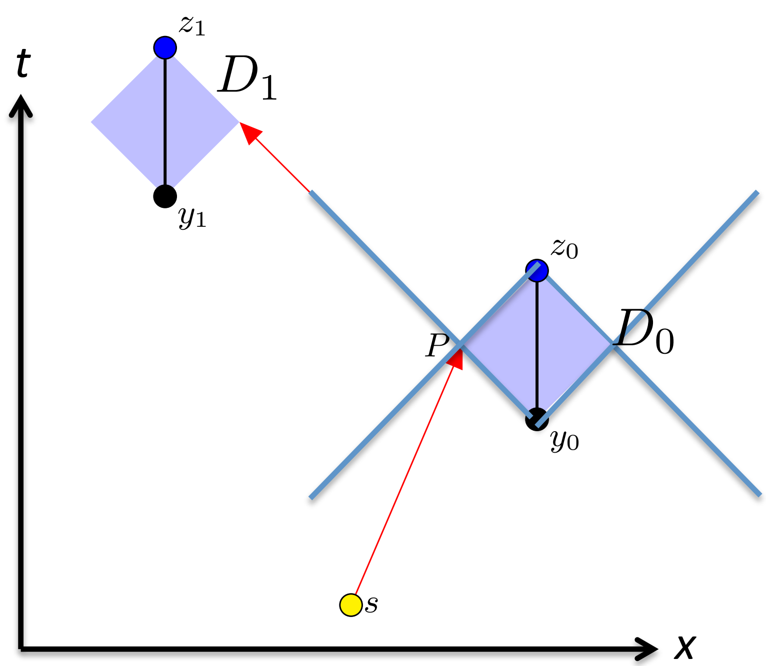
Teleportation measurement performed at s, with outcome forwarded to both z_0 and z_1

Half of Bell pair at y_0 sent to either z_0 or z_1 depending on request

Classical data: unconstrained by no-cloning
Entanglement: unconstrained by causality

Kent : arXiv:1204.4022

Summoning as replication



Define *causal diamond* D_j to be the intersection of the future of y_j and the past of z_j.

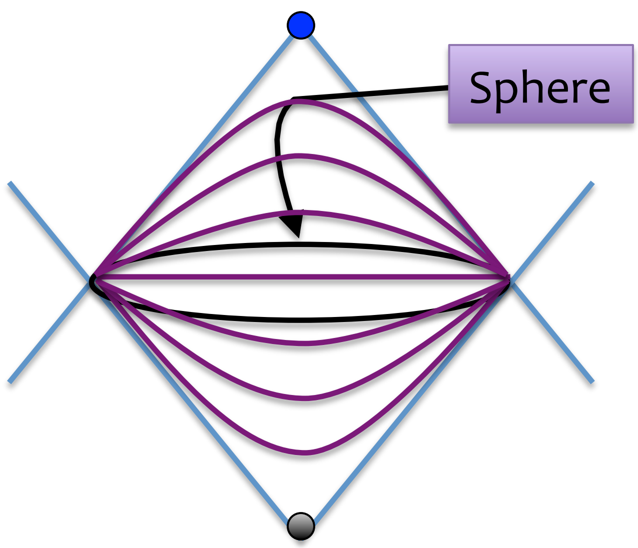
D_j consists of the points that can both be affected by the request at y_j and can affect the outcome at z_j.

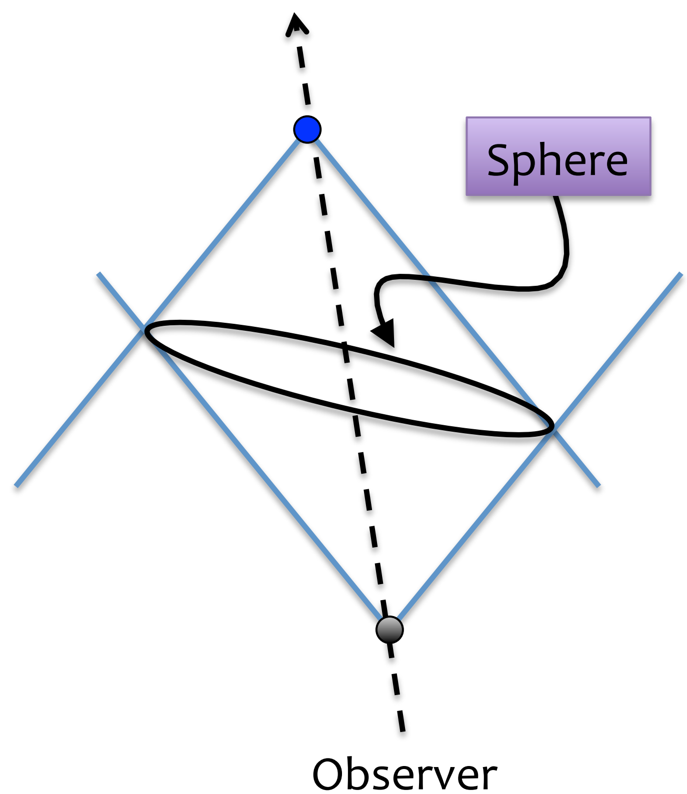
Summoning provides an operational definition of what it means for quantum information to be localized in the diamond D_j.

Summoning is possible iff z_1 is in the future of y_0 or z_0 is in the future of y_1.

Summoning is possible iff the causal diamonds D_0 and D_1 are *causally related*: there exists a causal curve from D_0 to D_1 or vice-versa.

Causal diamond geometry

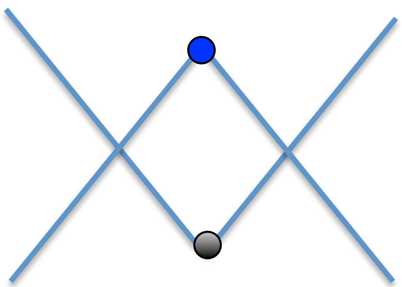


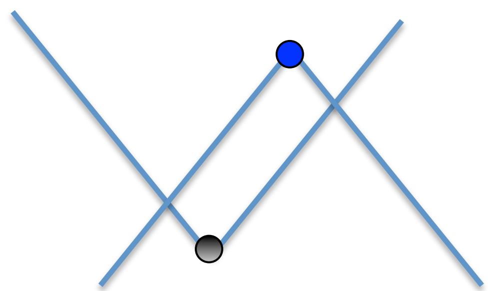


Inside a causal diamond, quantum mechanical time evolution is unitary

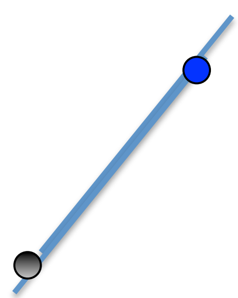
If φ is present on one spacelike slice, it is present on all of them

Causal diamond geometry

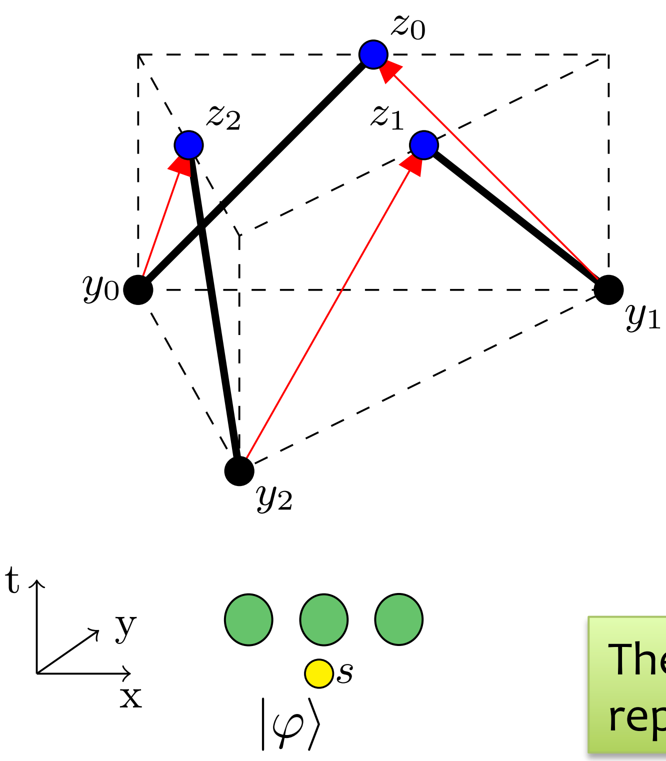




Diamond becomes a line segment when top and bottom are lightlike separated:



Exploiting quantum error correction



φ is encoded into ((2,3)) threshold quantum error correcting code at s

One particle sent to each of y_j

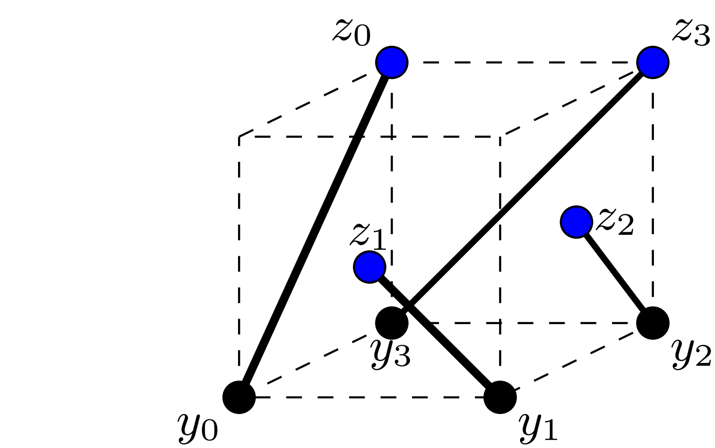
Each particle is then sent at the speed of light along a red ray

2 particles pass through each causal diamond $y_j z_j$

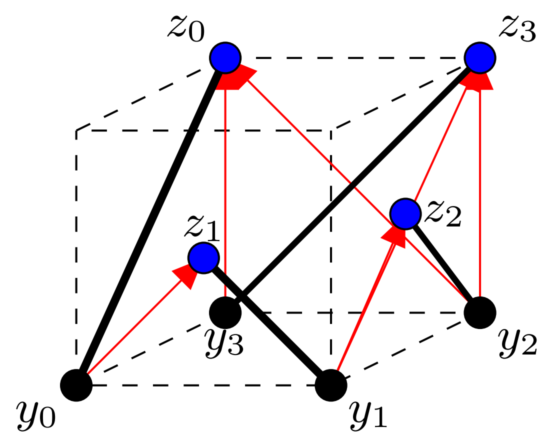
The same quantum information is replicated in each causal diamond

Summoning language: if a request is made at y_j then the share at y_j is sent to z_j instead of to z_{j-1}

A more complicated scenario



Can the same quantum info be replicated in all four diamonds?



Hrmm… Maybe? All diamonds are causally related

Yes! Each and every diamond can contain the same quantum information iff every pair is causally related

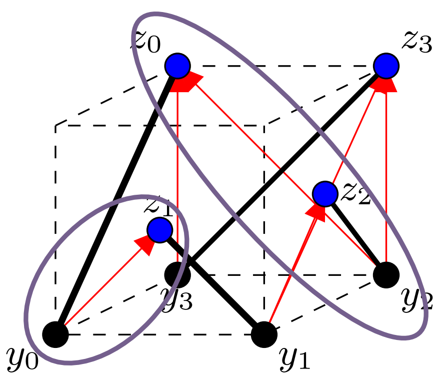
Equivalently: iff there is no *obvious* violation of causality or no-cloning

Information replication: general case

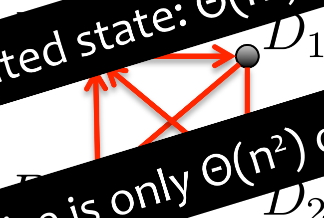
Each and every causal diamond can contain the same quantum information
if and only if every pair is causally related.

Proof: For n=2, the teleportation procedure works for any pair of diamonds

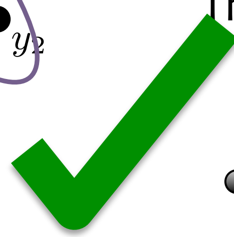
Assume the existence of a procedure for sets of n-1 diam…

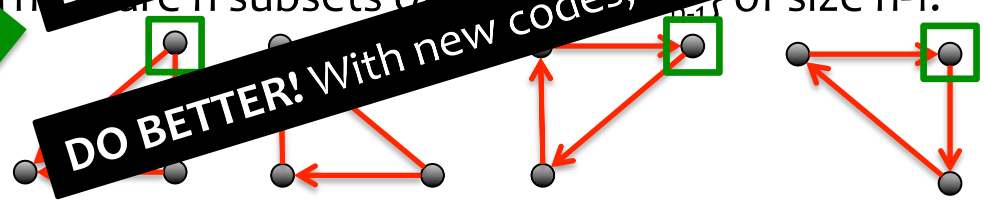


Graph of causal relations…



Th… re n subsets of {D… of size n-1.





BAD: Size of recursively generated state: Θ(n!) qubits

DO BETTER! With new codes, size is only Θ(n^2) qubits

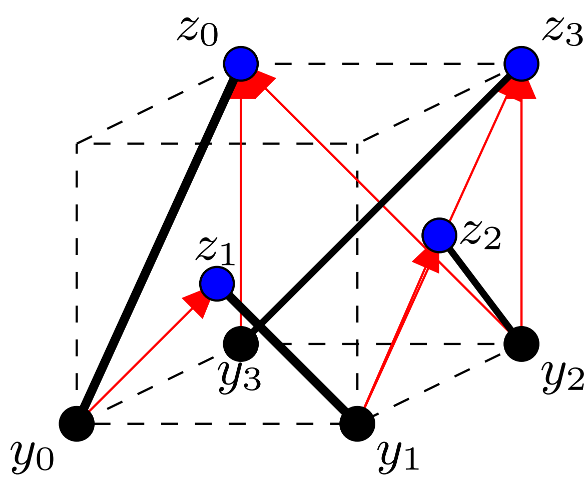
Request at y_1

z_1 receives n-1 "particles"

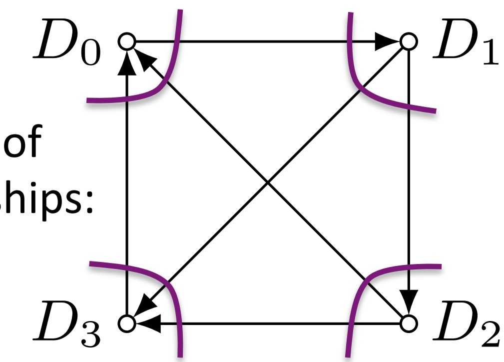
Encode φ into an ((n-1,n)) threshold quantum code.

Associate one "particle" to each such subset and for each subset execute the protocol recursively with one diamond removed.

Efficient procedure



G = (V,E) graph of causal relationships:



Encode φ into a quantum error correcting code with one share for each edge.

Code property: φ can be recovered provided all the shares associated to any D_j

Execute the n=2 teleportation protocol for each edge.

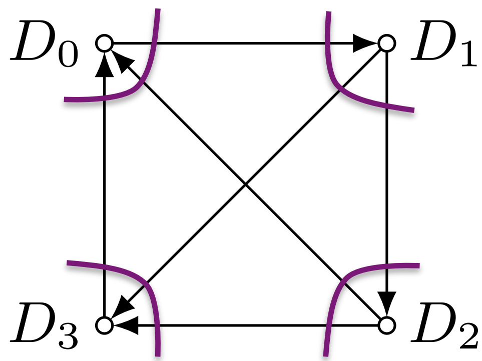
If request made at y_j, then z_j receives all shares associated to D_j and can recover φ.

Unusual QEC: ~n^2 qubits but recovery using n-1. Vanishing fraction O(1/n).

The quantum error correcting code

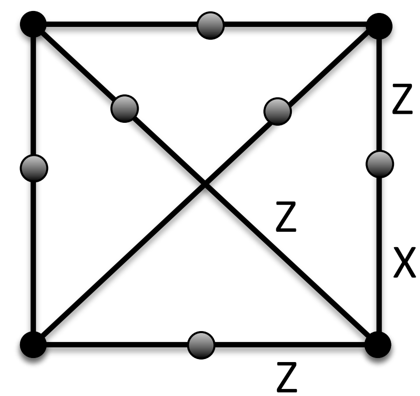
Designed using the codeword-stabilized (CWS) quantum code formalism [CSSZ'08]

G = (V,E)



Subdivide every edge:

G'=(V',E')



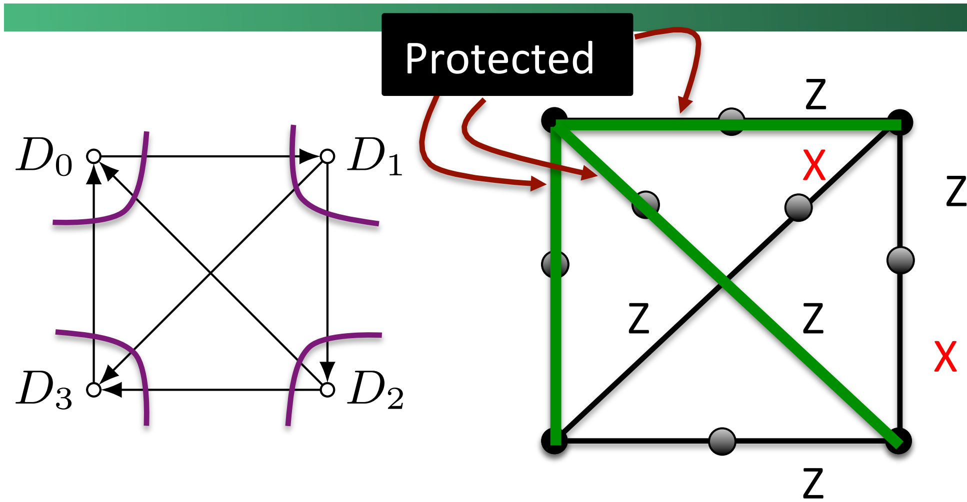
One qubit for each edge of G': |E'| = n(n-1)

Define commuting operators $S_e = X_e \prod_{f \in N_e} Z_f$

Code subspace is the span of the simultaneous +1 and -1 eigenspaces of all S_e.

Each share consists of the 2 qubits associated with each original edge of G.

Analysis of the code



CWS code property:
All errors converted to Z errors

For Pauli error P:
Err(P) = induced Z error

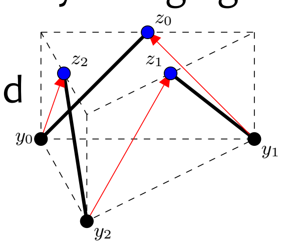
Condition 1: $\text{Err}(P) \neq \prod_e Z_e$

Condition 2: If $\text{Err}(P) = I$ then $[\prod_e Z_e, P] = 0$.

- Every possible X error induces exactly one Z error on a green edge
- To achieve Err(P) = I, need an *even* number of X errors.
- XZ=-ZX implies that if P contains an even number of X errors, then $[\Pi_e Z_e, P]=0$

Conclusions

- Quantum information can be replicated in a surprising variety of ways in spacetime
- Only constraints: no obvious violations of no-cloning or causality
- Straightforward extension to arbitrary spatial regions
- Similar ideas can also be used to **exclude** information from specified regions
  - Previously studied at fixed time as "quantum secret sharing"
  - Extends theory to dynamically changing coalitions and moving participants
- Future directions:
  - Convince someone to build



  - Incorporate further physical constraints
  - Systematic theory of information processing in relativistic spacetime!
  - Extract lessons for situations in which spacetime structure is an approximation:
    - Cloning paradoxes in black hole evaporation, complementarity, firewalls, etc.
    - How do conclusions change when area scaling of qubits is required?